

eQE: The embedded Quantum ESPRESSO software package

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MolSSI workshop at UB 2018

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Est. 2012

Embedding is the future present

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The playing field as I see it

- Moore's law forces us to go parallel
- (Super)computer architectures typically provide
 - Compute nodes (12-24 procs/node)
 - Infiniband
 - Coprocessors
- The cloud (*probably the future*)
 - Low on CPU, high on GPU
 - Google colaboratory / Google cloud: python + tensorflow
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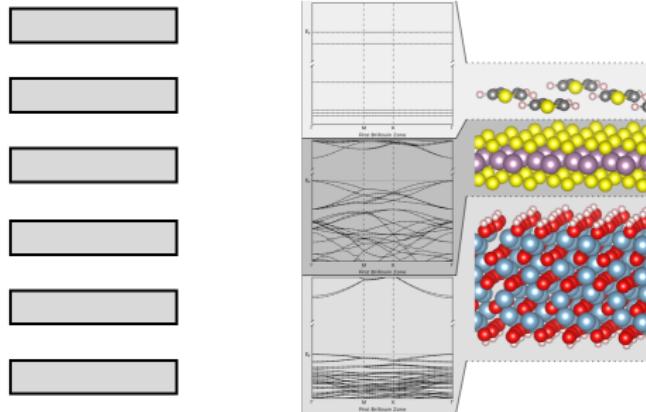
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The role of Embedding

- Naturally work-parallel
- In some formulations also data-parallel
- Embedding is a paradigm:
 - Can embed mathematical quantities (ρ , Ψ , G)
 - Can embed QC/CMT codes

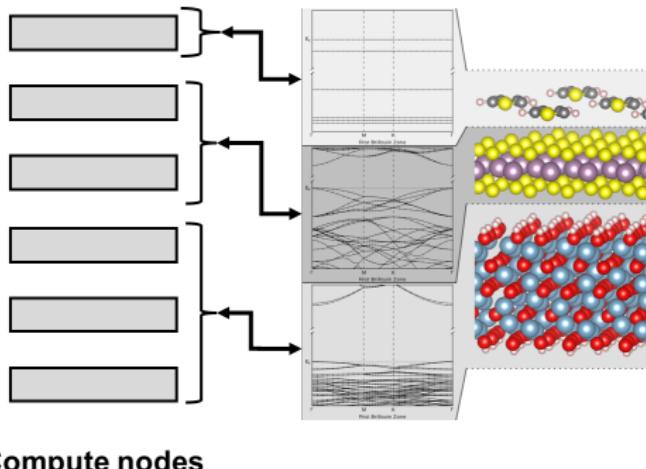
Embedding overlaps with current architectures

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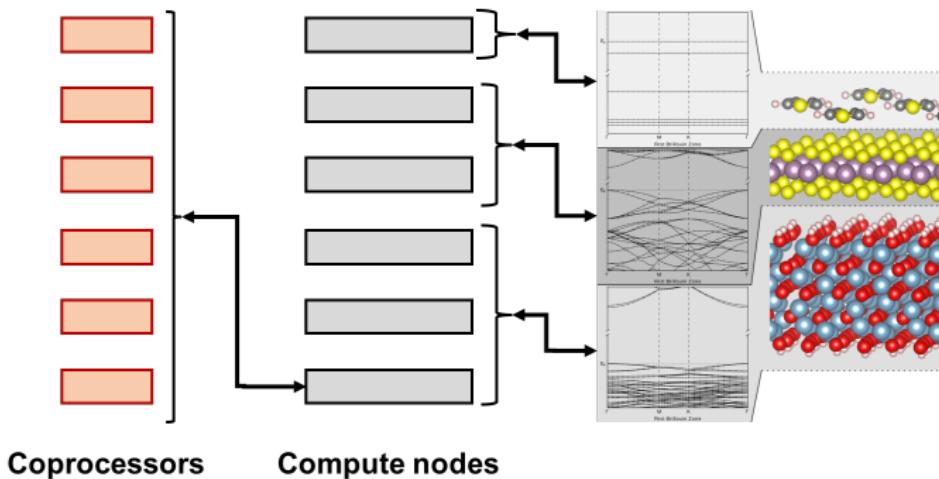


Compute nodes

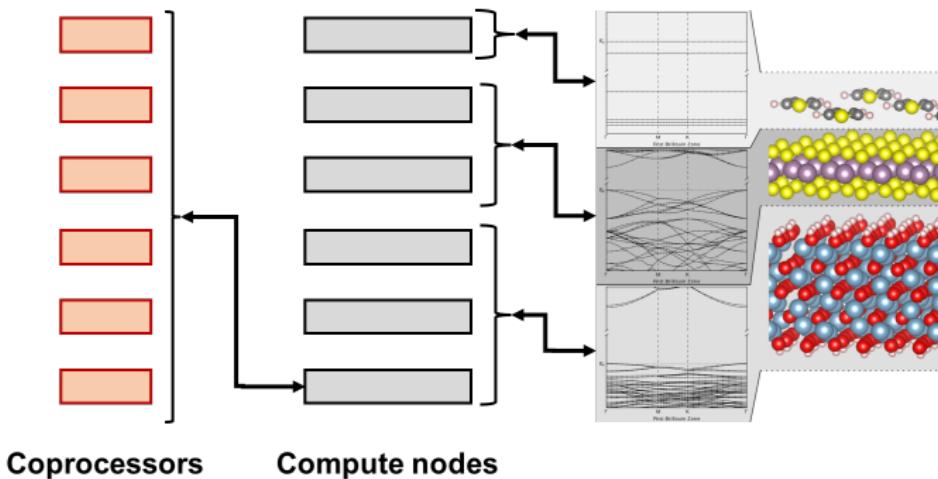
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The method should:

- Broadcast as little data as possible across compute nodes
- Avoid computing zeros
 - Simple and easy basis set truncation
- Approach excited states and $e^- - N$ dynamics

Density Embedding (FDE) / Subsystem DFT

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- **Frozen Density Embedding** (FDE): Coupled Kohn–Sham equations for each subsystem

$$\frac{\delta E_{\text{FDE}}[\rho_I + \rho_{II}]}{\delta \rho_I} = 0 \rightarrow \left[-\frac{1}{2} \nabla^2 + v_{KS}^I(\mathbf{r}) + v_{emb}^I(\mathbf{r}) \right] \phi_{(i)_I}(\mathbf{r}) = \varepsilon_i^I \phi_{(i)_I}(\mathbf{r})$$

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Orbital-free DFT interaction between subsystems

... a slide from 2018 QE workshop...

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considering that $\rho(r) = \rho_I(\mathbf{r}) + \rho_{II}(\mathbf{r})$.

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Homework :)

- Compute $T_s^{\text{nadd}}[\rho_I, \rho_{II}]$ in the Thomas-Fermi approximation,
 $T_s[\rho] = C_{TF} \int \rho^{5/3}(\mathbf{r}) d\mathbf{r}$
- Compute $E_x^{\text{nadd}}[\rho_I, \rho_{II}]$ in the Dirac approximation, $E_x[\rho] = C_x \int \rho^{4/3}(\mathbf{r}) d\mathbf{r}$
- Compute $E_H^{\text{nadd}}[\rho_I, \rho_{II}]$, $E_H[\rho] = \frac{1}{2} \int \int \rho(\mathbf{r}) \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}'$

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... more homework!

- T_s contribution to $v_{emb}(\mathbf{r})$
- E_x contribution to $v_{emb}(\mathbf{r})$
- E_H contribution to $v_{emb}(\mathbf{r})$

eQE — embedded Quantum Espresso

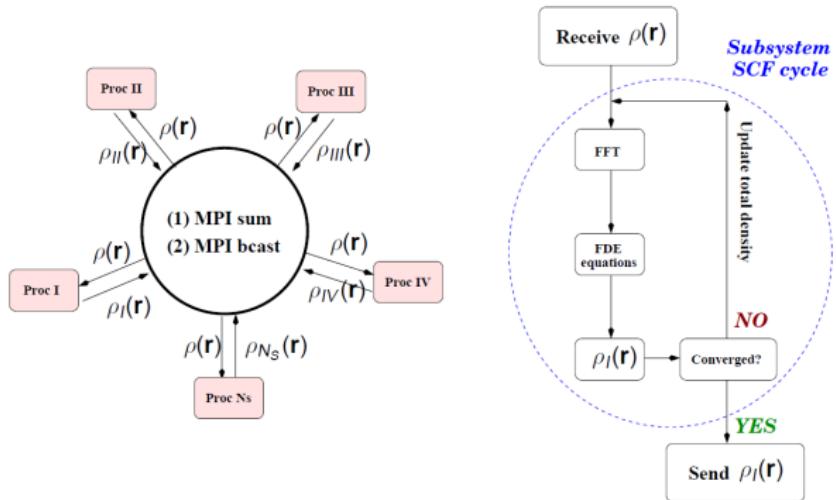


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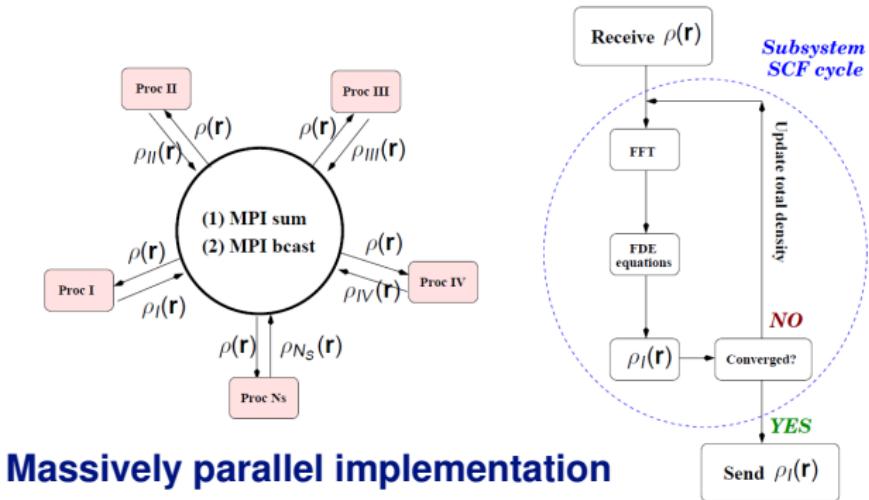
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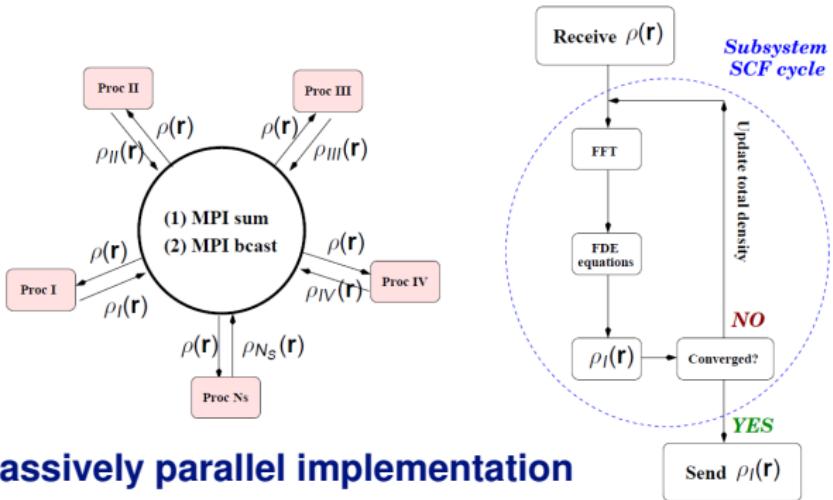


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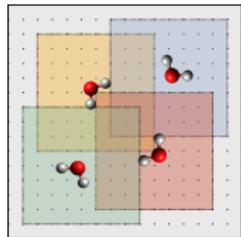
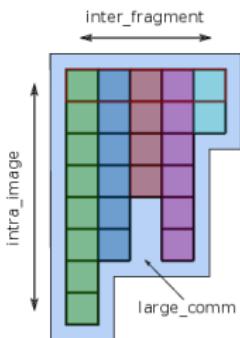


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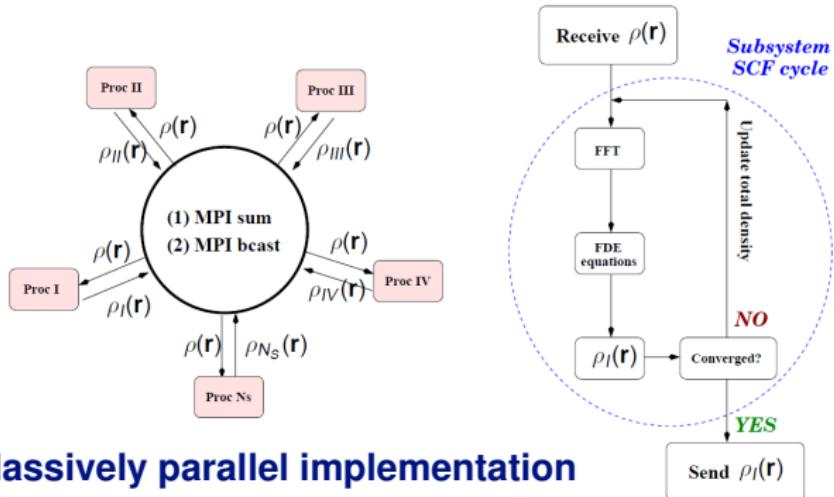
Rewrite the MPI module of QE

- Subsystem-specific # of CPUs
- Improved latencies (processes wait for others to complete)
- Nested DIIS for $\{\rho_I(\mathbf{r})\}$





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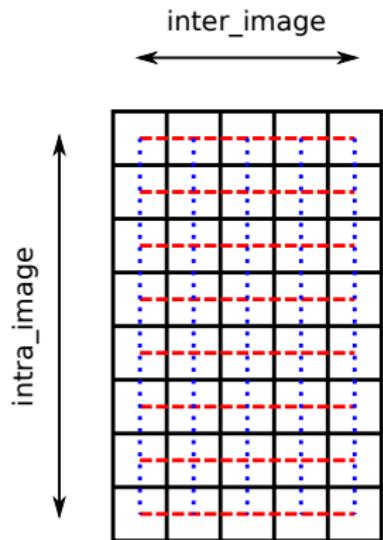
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BZ sampling (k -points)

- K -point sampling for (semi)conductors.
- Γ -point for molecules/insulators.

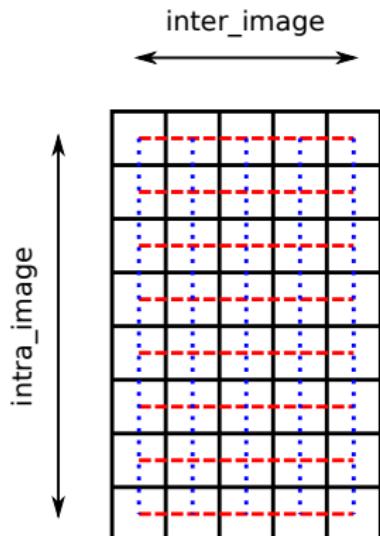
eQE: a note on parallelization

Regular QE

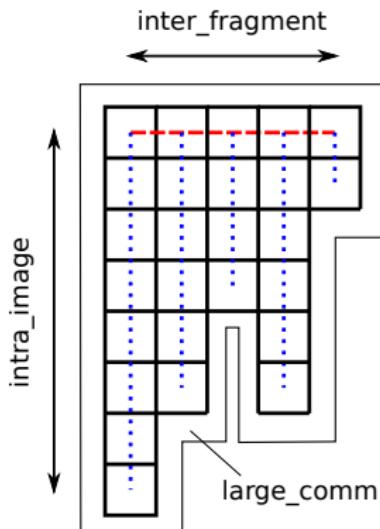


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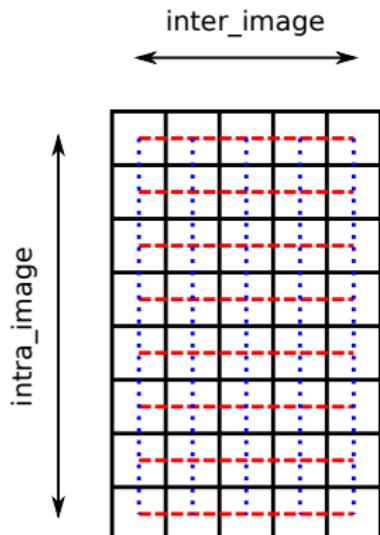


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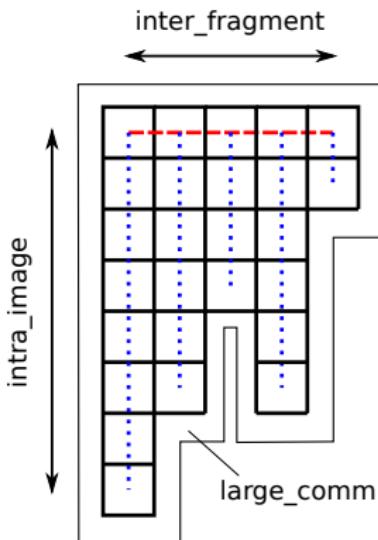


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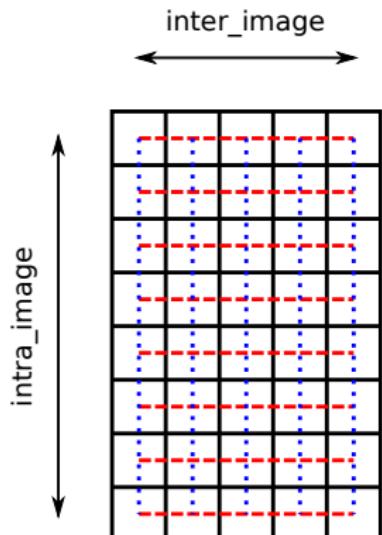


pros & cons

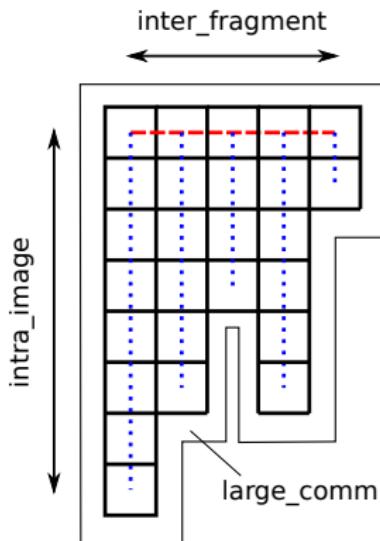
- distributed data communication
- non-polymorphic

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eQE



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- polymorphic
- gathered data communication

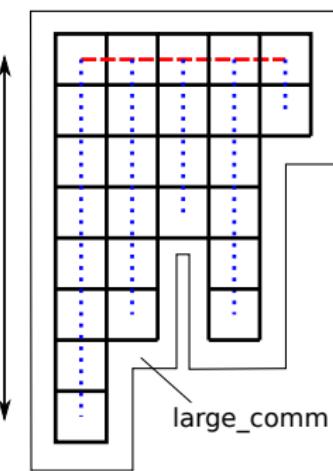
eQE: a note on coding it

eQE

inter_fragment



intra_image



```
! dfftp          :: the fft descriptor of the subsystem electron cell
! diffp          :: the fft descriptor of the supersystem simulation cell
! rho%of_r(dfftp%nnr)   :: the subsystem density in the subsystem cell
                           :: (grid is distributed over the processes in intra_image_comm)
! rho_fde%of_r(dfftp%nnr) :: the supersystem density in the subsystem cell
                           :: (grid is distributed over the processes in intra_image_comm)
! rho_fde_large%of_r(dfftl%nnr) :: the supersystem density in the supersystem cell
                           :: (grid is distributed over the processes in large_comm)
                           :: same quantity in reciprocal space
! %of_g(:)        :: auxiliary vector of the subsystem cell real space
! aux(dfftp%nnr)  :: (grid is distributed over the processes in intra_image_comm)
! gaux(ngm)       :: auxiliary vector of the subsystem cell reciprocal space
! rauxx(nr1*nr2*nr3) :: subsystem cell real space auxiliary vector
! auxlx(dfftl%nnr) :: (whole grid is collected on the ionode proc of intra_image_comm)
! gauxl(ngml)     :: auxiliary vector of the supersystem cell real space
! rauxl(nr1*nr2l*nr3l) :: (grid is distributed over the processes in large_comm)
                           :: auxiliary vector of the supersystem cell reciprocal space
                           :: supersystem (large) cell real space gathered auxiliary vector
                           :: (whole grid is collected on the ionode proc of large_comm)
! f2l(nr1*nr2*nr3) :: subsystem cell to supersystem cell mapping vector.
```

Collect the subsystem density
from all the intra_image
processes into a single array
(raux).

Copy each subsystem density
into an array of the
supersystem cell (rauxl), using
mapping (f2l).

Allreduce rauxl across the
processes in inter_fragment
to build the supersystem
density in the supersystem cell.

Using the f2l mapping, copy
the supersystem density into
each subsystem cell.

Distribute the supersystem
density over all processes in
the communicators spanning
the supersystem and
subsystem cells.

FFT to obtain the reciprocal
space representation of the
supersystem density in both
the subsystem and the
supersystem cells.

```
call grid_gather(rho_fde%of_r(:,is), raux)
```

```
if (ionode) then
```

```
    rauxl = 0.d0
    rauxl(f2l(:)) = raux(:)
```

```
call mp_sum(rauxl, inter_fragment_comm)
```

```
raux = 0.d0
raux(:) = rauxl(f2l(:))
```

```
endif
```

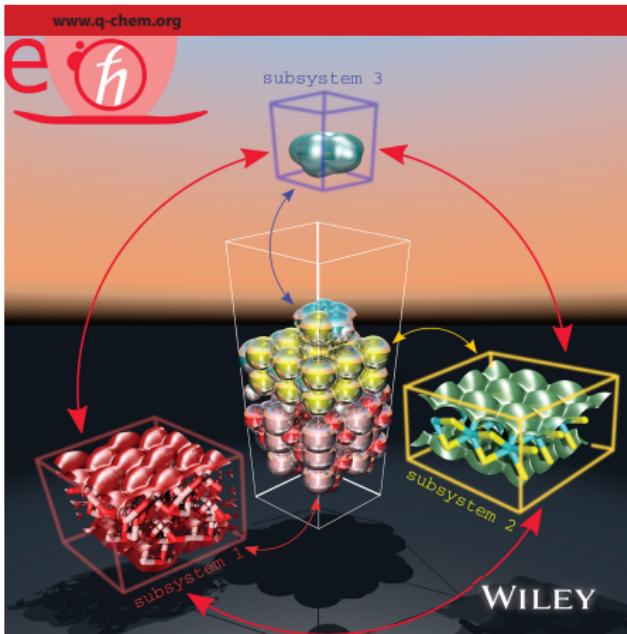
```
call grid_scatter_large(raux, rho_fde_large%of_r(:,is))
call grid_scatter(raux, rho_fde%of_r(:,is))
```

```
gauxl(:) = cmplx(rho_fde_large%of_r(:,is), 0.d0, kind=dp)
call fwfft ('Custom', gauxl, dfftl)
rho_fde_large%of_g(l:ngml,is) = gauxl(nll(1:ngml))
```

```
gaux(:) = cmplx(rho_fde%of_r(:,is), 0.d0, kind=dp)
call fwfft ('Dense', gaux, dfftp)
```

eQE — embedded Quantum Espresso

International Journal of QUANTUM CHEMISTRY



eQE: An open-source density functional embedding theory code for the condensed phase
International Journal of Quantum Chemistry, **117**, e25401 (2017)

eQE Development: Hackathons are the key

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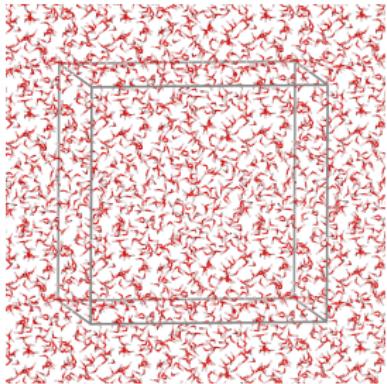
eQE Development: Hackathons are the key



eQE: Performance for molecular periodic systems

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Water 1024



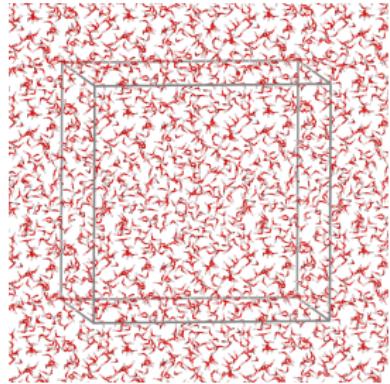
30726 \AA^3

Speedup compared to regular QE (all PBE)

24.5 ×

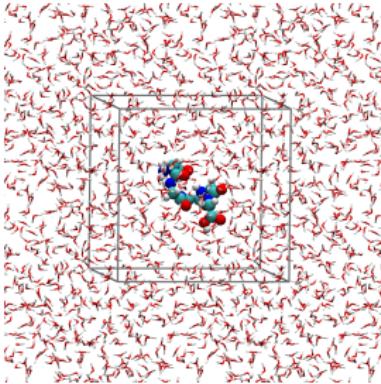
eQE: Performance for molecular periodic systems

Water 1024



30726 \AA^3

(GLY)₆ in (H₂O)₃₉₅



12656 \AA^3

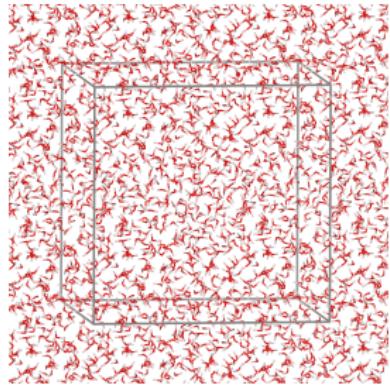
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36.5 ×

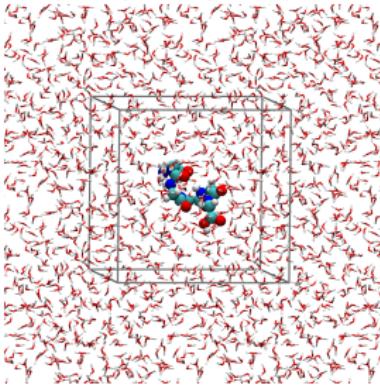
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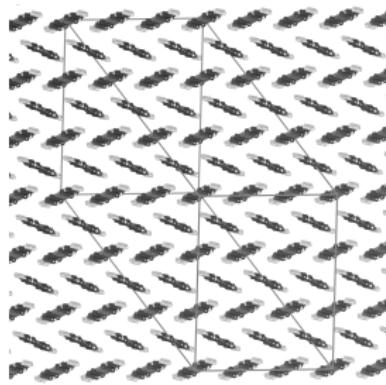
30726 Å³

(GLY)₆ in (H₂O)₃₉₅



12656 Å³

Pentacene 3 × 3 × 3



18243 Å³

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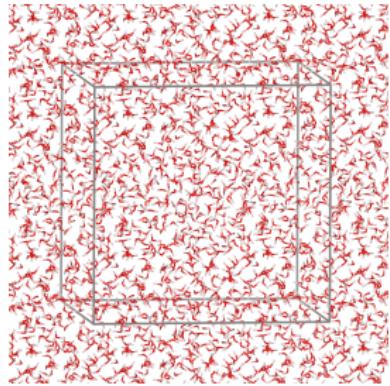
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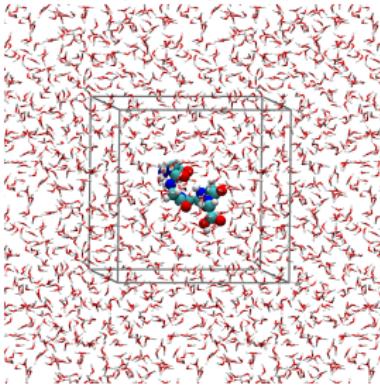
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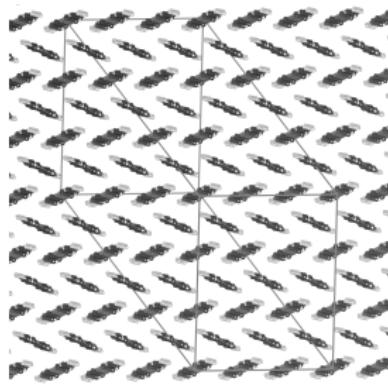
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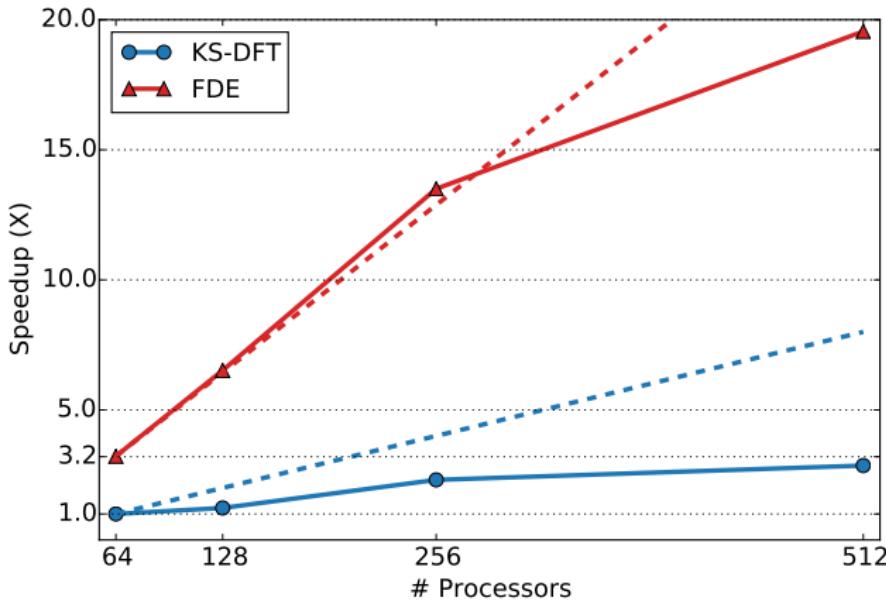
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Well over one order of magnitude speedup!

eQE vs QE: Parallel scaling for water 256

Water 256, 256 subsystems

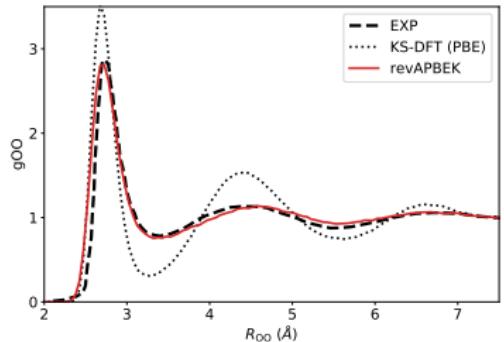


QE–eQE gap widens with increasing # of CPUs

Liquid Water – does eQE get the structure right?

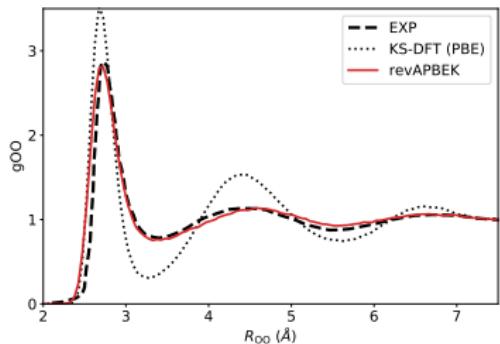
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Oxygen–Oxygen RDF

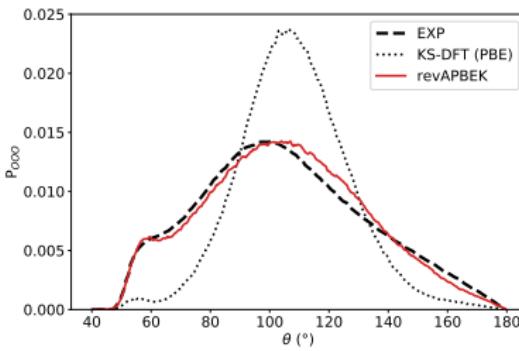


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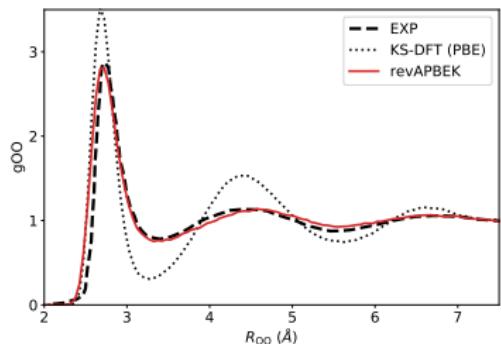


O–O–O ADF

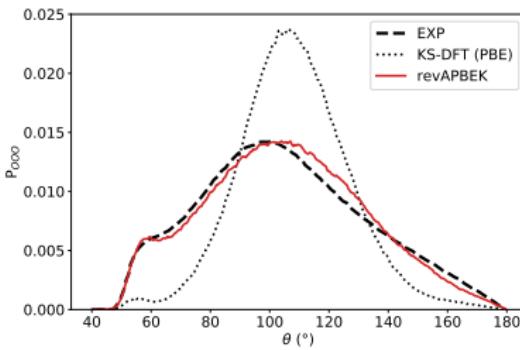


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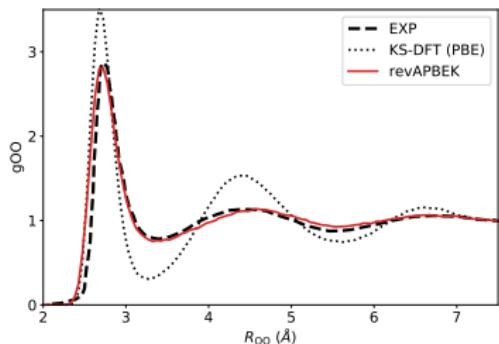
Diffusion coefficient and dipole moment

$$\langle D \rangle = 2.97(0.4) \cdot 10^{-5} \text{ cm}^2 \text{s}^{-1}$$

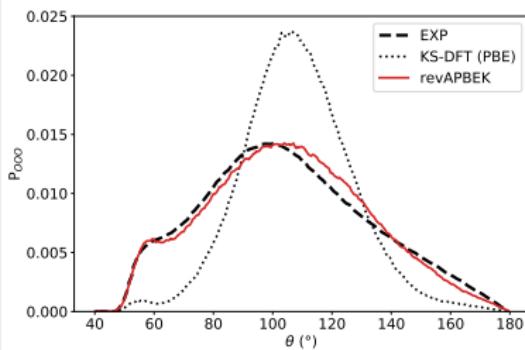
$$\langle \mu \rangle = 2.8(0.2)D$$

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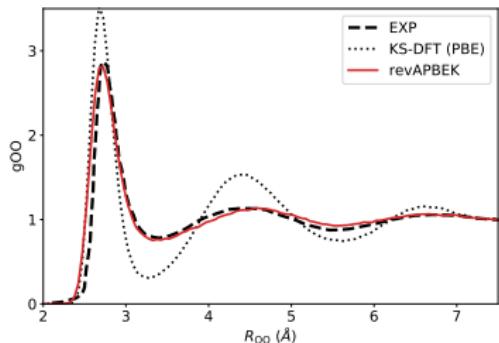
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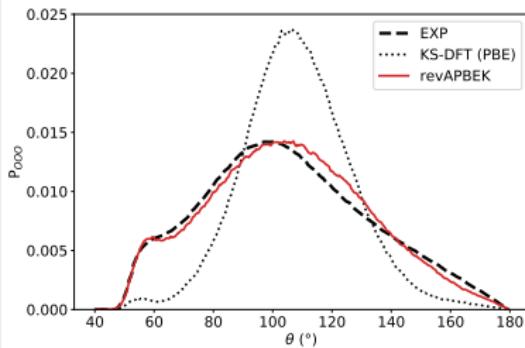
- eQE recovers correct structure
- eQE recovers correct dynamics
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Liquid Water – does eQE get the structure right?

Oxygen–Oxygen RDF



O–O–O ADF



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- **How about e^- excited states?**

Subsystem TDDFT and Related Methods

- effective excitations of active subsystem in environment (leads to "FDEu")
M.E. Casida, T.A. Wesolowski, *Int. J. Quant. Chem.* **96** (2004), 577; T.A. Wesolowski, *JACS* **126** (2004), 11444.
- general subsystem TDDFT formulation for delocalized excitations ("FDEc")
J. Neugebauer, *J. Chem. Phys.* **126** (2007), 134116; J. Neugebauer, *J. Chem. Phys.* **131** (2009), 084104
- derivation with focus on subsystem response functions
M. Pavanello, *J. Chem. Phys.* **138** (2013), 204118.
- fragment-based TDDFT in the context of partition DFT
M.A. Mosquera, D. Jensen, A. Wasserman, *Phys. Rev. Lett.* **111** (2013), 023001.
- time-dependent potential-functional theory for subsystems
C. Huang, F. Libisch, Q. Peng, E.A. Carter *J. Chem. Phys.* **140** (2014), 124113.
- real-time subsystem TDDFT
A. Krishtal, D. Ceresoli, M. Pavanello, *J. Chem. Phys.* **142** (2015), 154116.
- subsystem TDDFT with external orthogonality
D.V. Chulhai, L. Jensen, *Phys. Chem. Chem. Phys.* **18** (2016), 21032.

Subsystem TDDFT: *Response functions*

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$$\delta\rho(\mathbf{r}, \omega) = \sum_I \delta\rho_I(\mathbf{r}, \omega) \quad \text{Derives directly from } \rho(\mathbf{r}) = \sum_I \rho_I(\mathbf{r})$$

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The K_{IJ} coupling

$$K_{IJ}(\mathbf{r}_1, \mathbf{r}_2, t - t') = \frac{\delta(t - t')}{|\mathbf{r}_1 - \mathbf{r}_2|} + \frac{\delta^2 E_{xc}}{\delta\rho(\mathbf{r}_1, t)\delta\rho(\mathbf{r}_2, t')} + \frac{\delta^2 T_s}{\delta\rho(\mathbf{r}_1, t)\delta\rho(\mathbf{r}_2, t')}$$

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Subsystem TDDFT is a general theory for open systems
Let's apply it to liquid water!

The Optical Spectrum of Liquid Water

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*Cooperation and Environment Characterize the Low-Lying Optical Spectrum
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J. Phys. Chem. Lett., 8 (20), pp 5077-5083 (2017)

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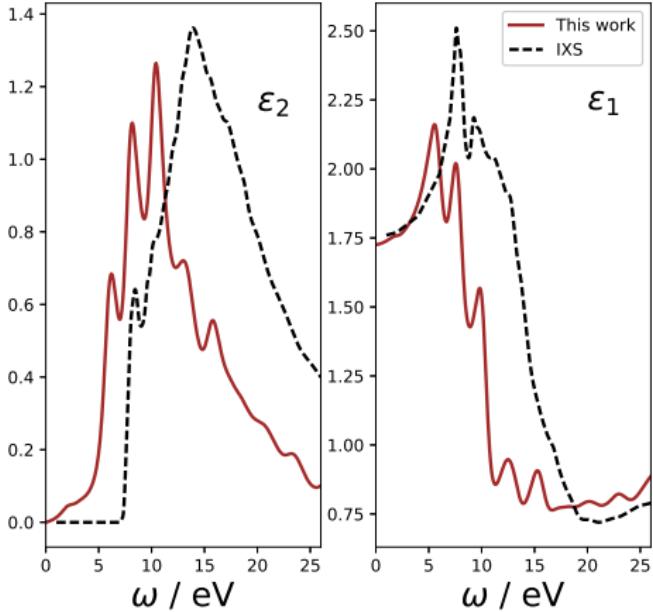
Optical Properties of Liquid Water: ϵ_1 & ϵ_2

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- 10 snapshots of water 64
- Average of 640 spectra

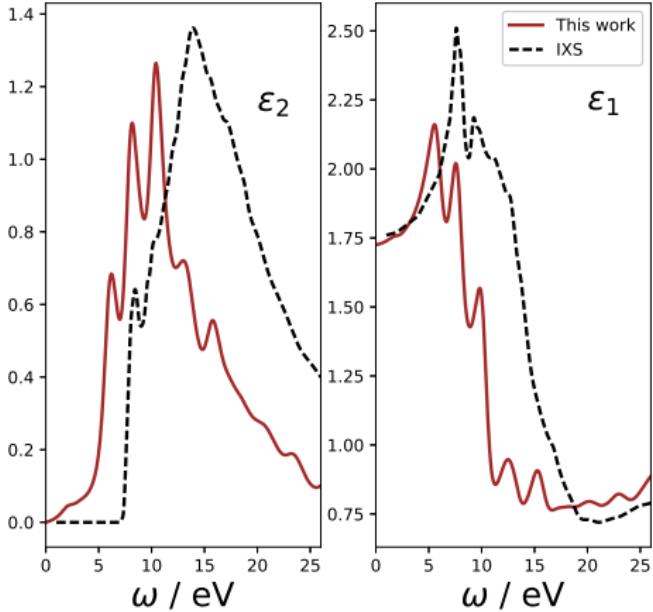


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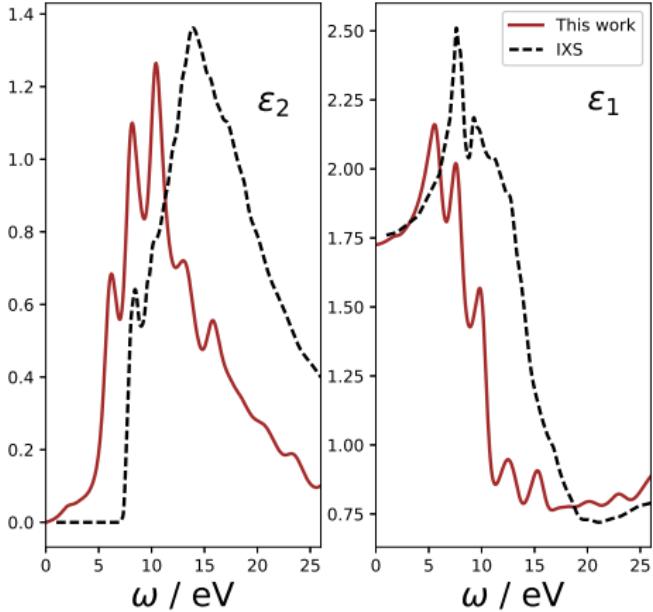
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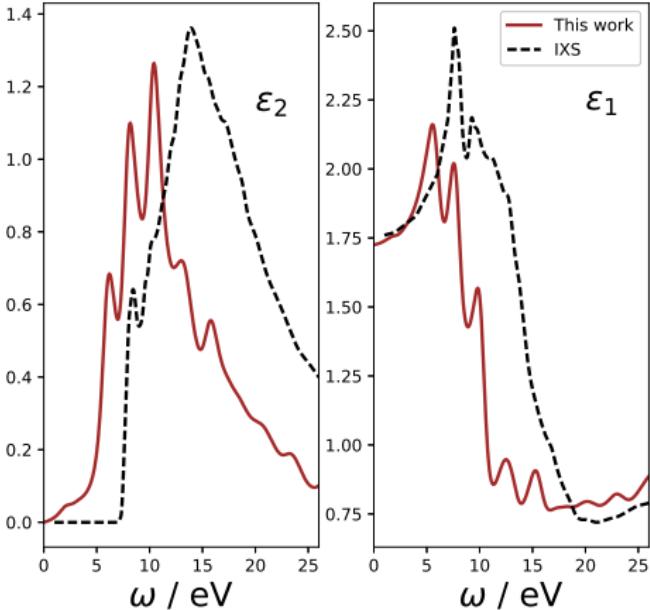
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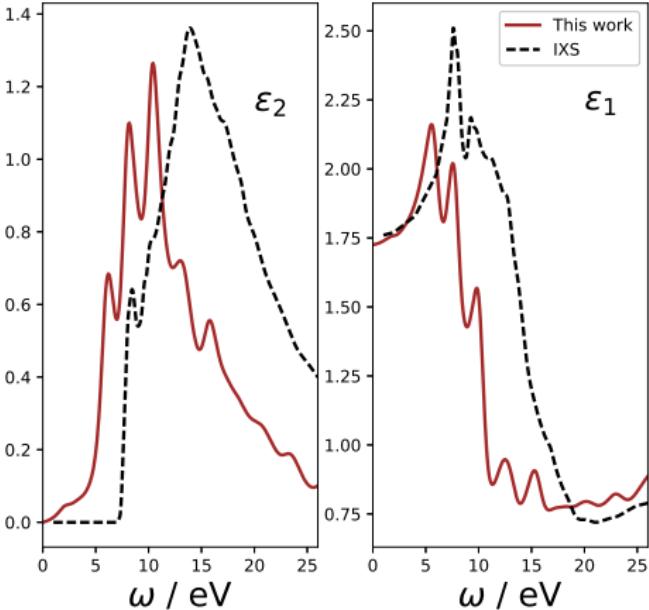
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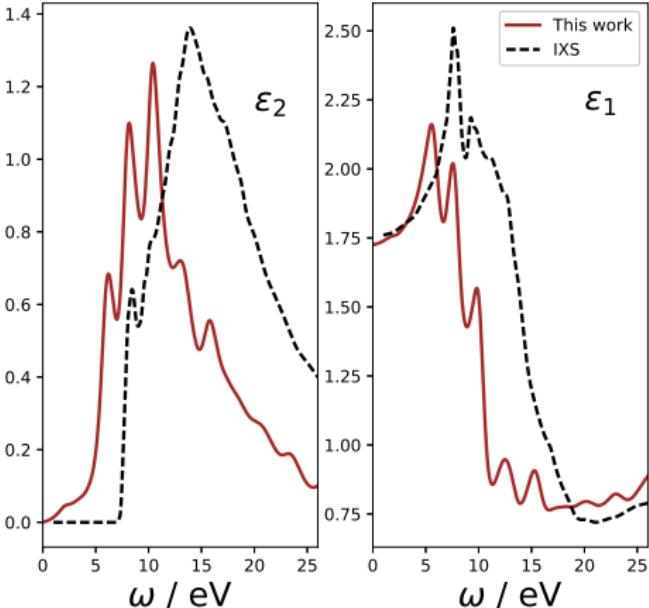
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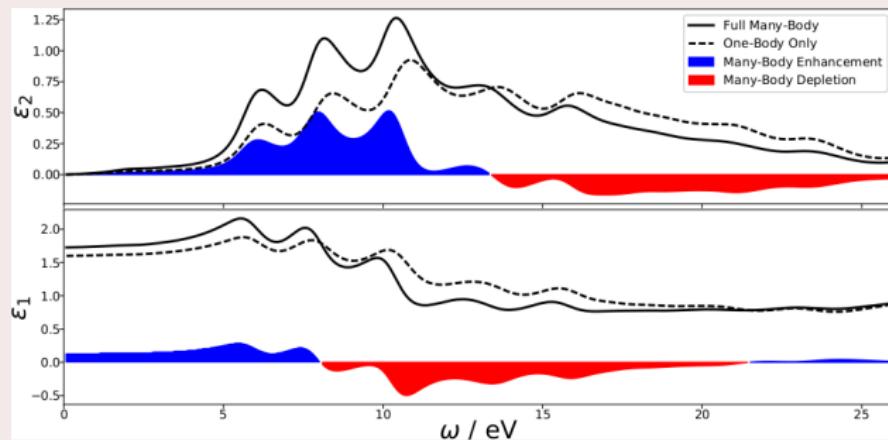
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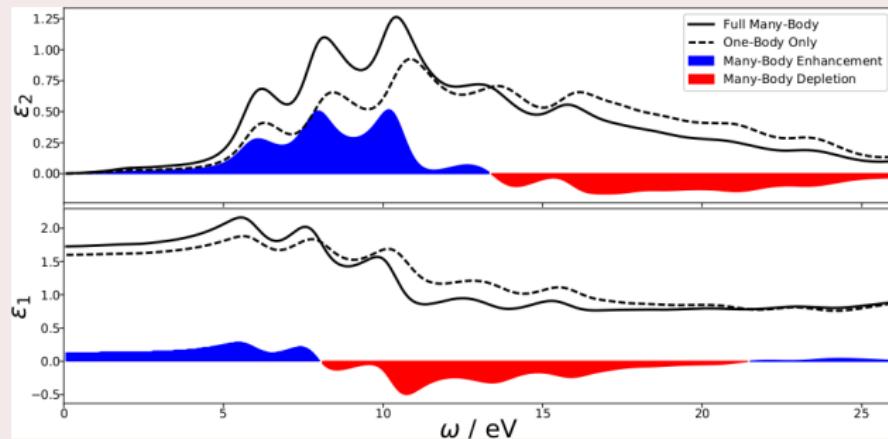
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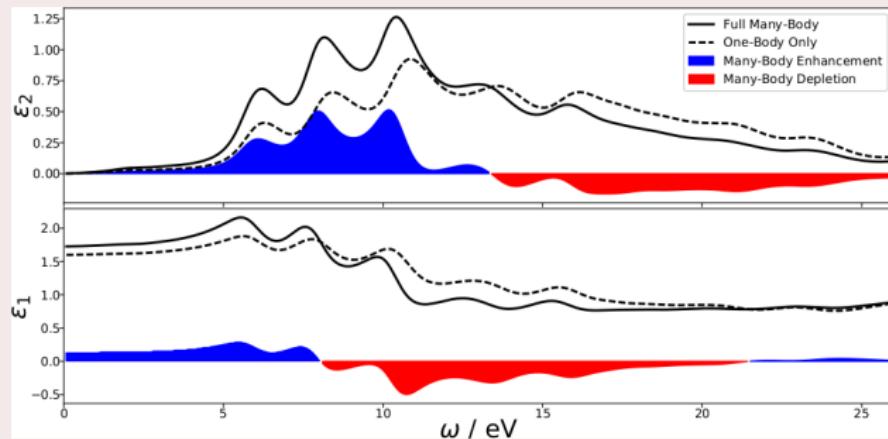
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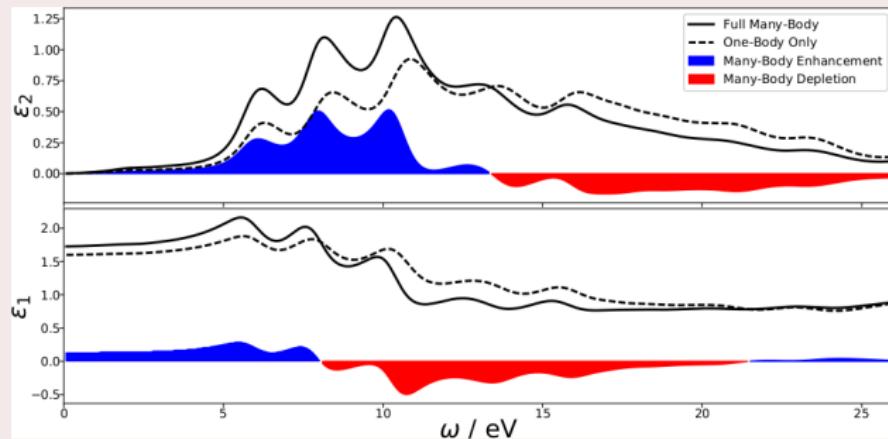
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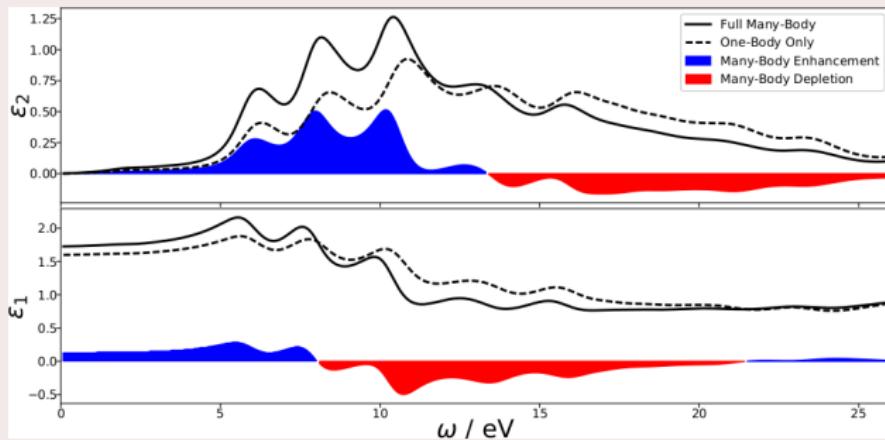
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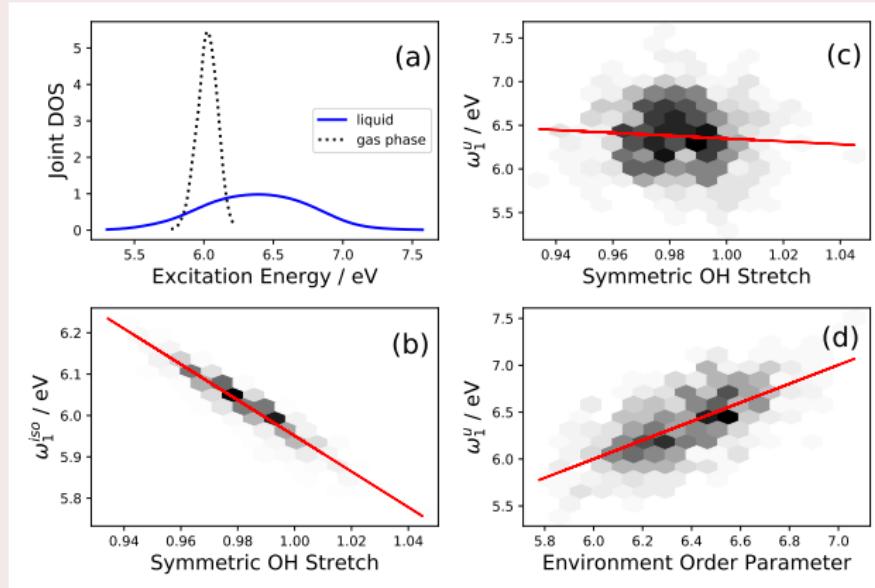
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First Absorption Band of Liquid Water: ω_1

Role of the Environment: Urbach tail and ω_1 peak position

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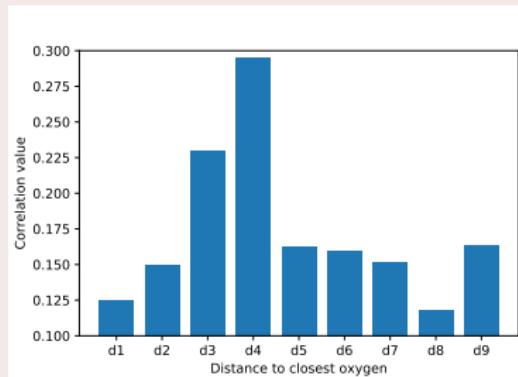
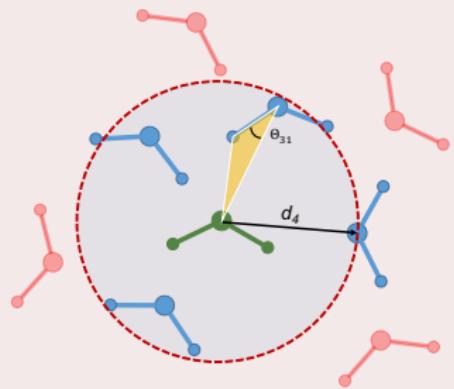
- Urbach tail entirely due to environment
- ω_1 and OH stretching do not correlate in the liquid

- Environment Order Parameter (**EOP**): Regression of 300+ descriptors
- Correlation of EOP to ω_1 is 0.65

Optical Properties of Liquid Water

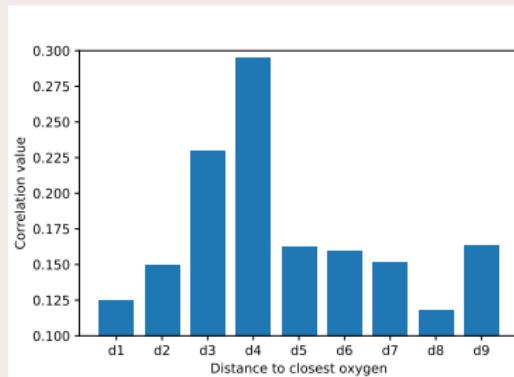
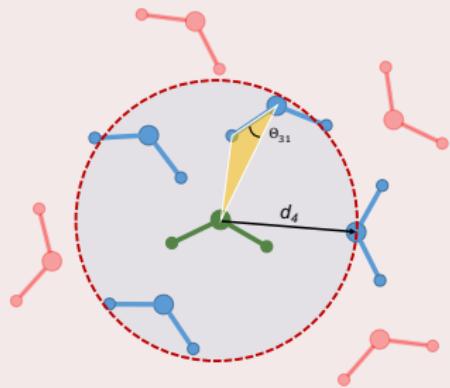
Optical Properties of Liquid Water

Environment Order Parameter: Simplified depiction



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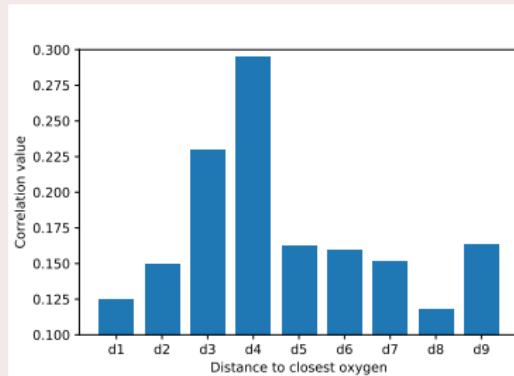
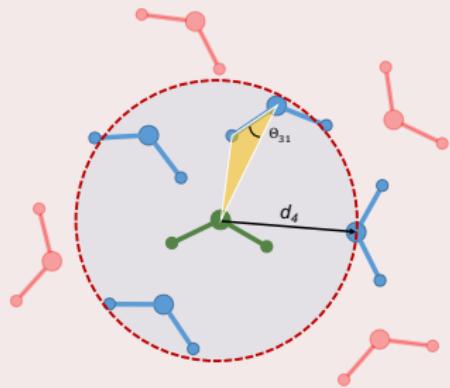
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What is “Environment Order Parameter” made of?

Optical Properties of Liquid Water

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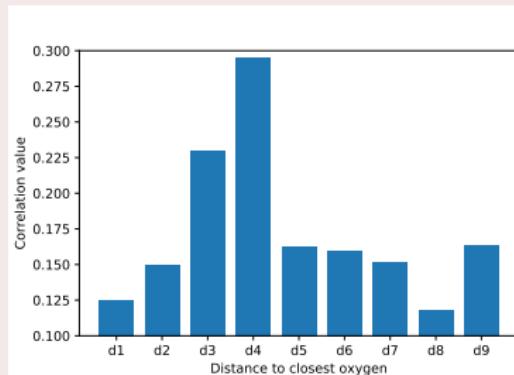
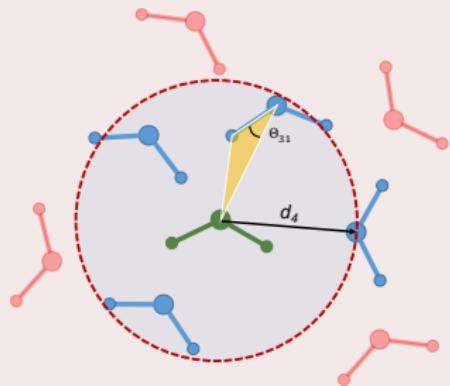


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- Accepted Hydrogen Bonds is the highest contribution to EOP

Optical Properties of Liquid Water

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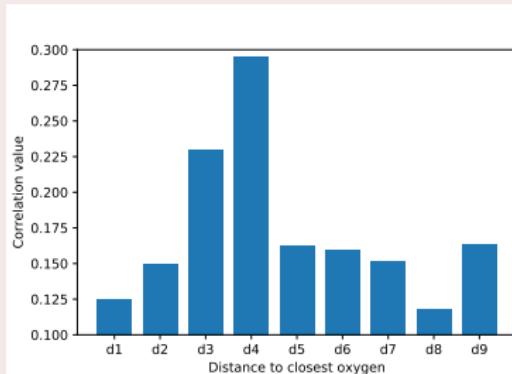
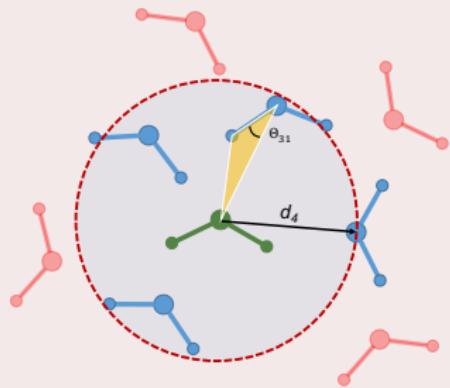


What is “Environment Order Parameter” made of?

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- Distances and angles within the 1st solvation shell

Optical Properties of Liquid Water

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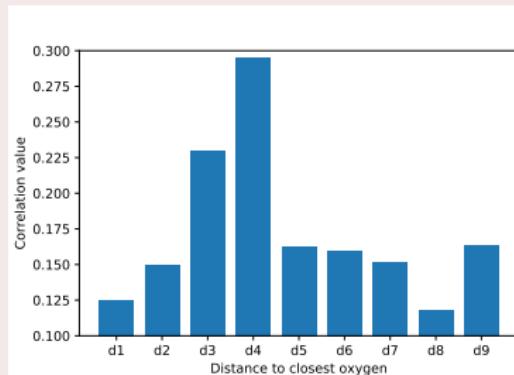
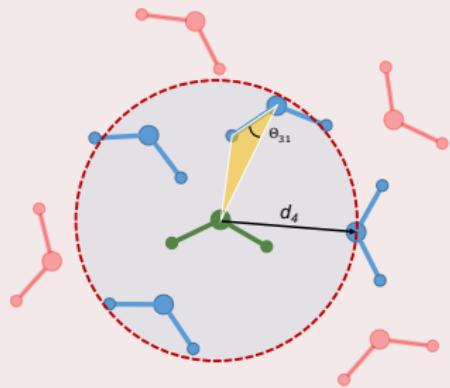


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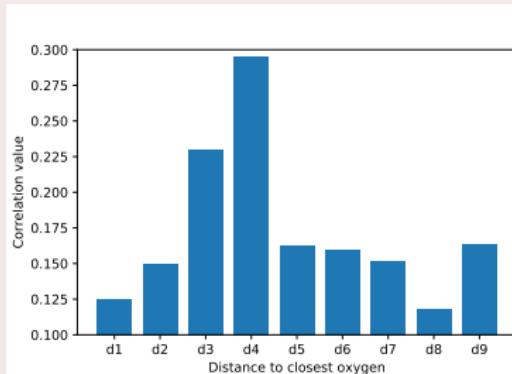
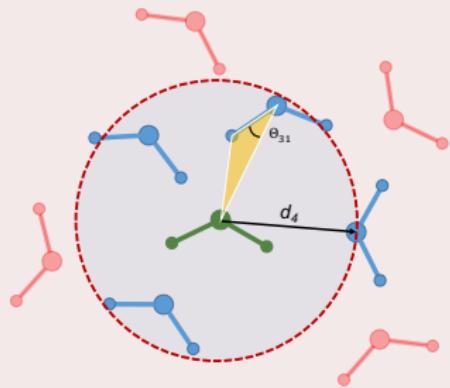


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... we have dissected liquid water...

Acknowledgments

Postdocs, Students & Collaborators

Current PRG members

Postdocs:

- Dr. Wenhui Mi
- Dr. Muhammed Acikgoz
- Dr. Pablo Ramos

Graduate Students:

- Rupali Chawla
- Alina Umerbekova
- Jack Maranhao

Funding:

- NSF CAREER
- DOE CTC

Alumni & Collaborators

Alumni:

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- **Dr. Alessandro Genova (@ Kitware)**
- **Prof. Alisa Krishtal (@ NJIT)**
- **Dr. Debalina Sinha (@ L'Oreal)**

Collaborators:

- **Dr. Davide Ceresoli (CNR)**
- **Prof. Rob DiStasio (Cornell)**
- **Dr. Andre Gomes (CNRS)**
- **Prof. Oliviero Andreussi (North Texas)**
- **Prof. Henk Eshuis (Montclair State)**
- **Dr. Damien Riedel (Paris Sud)**