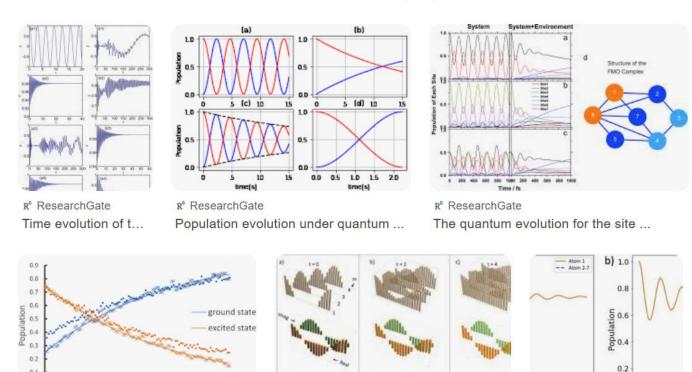


How to turn your on-the-fly dynamics code into a simulator of nonlinear spectroscopic signals via doorway-window method

Maxim F. Gelin



We, theoreticians, like to calculate population dynamics ...



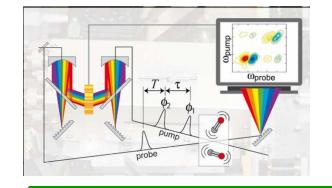
What's about spectroscopic signals?

chips 2584

1000

Time (picosecond)

4 chips 2452





How to turn your pump-probe instrument into a multidimensional spectrometer: 2D IR and Vis spectroscopies *via* pulse shaping

Sang-Hee Shim and Martin T. Zanni*

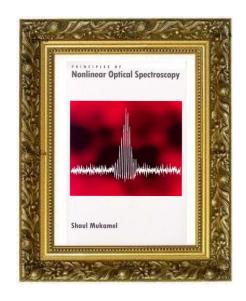
Received 12th August 2008, Accepted 30th October 2008

First published as an Advance Article on the web 10th December 2008

DOI: 10.1039/b813817f

I decided to plagiarize/modify Zanni's title:

How to turn your on-the-fly dynamics code into a simulator of nonlinear spectroscopic signals via doorway-window method





Fundamental notions and models (1995)

Computer revolution in theoretical spectroscopy

- AI & ML & Quantum Computers
- Ab initio parameterization of system Hamiltonians
- The use of more realistic (therefore complex and multidimensional) models of molecular systems (**HEOM and MCTDH are the game changers**)
- Simulation of spectroscopic signals on-the-fly



Before:

efficient analytical methods in many-body quantum dynamics

- Memory function formalism, Mori formalism
- Projection operator formalism
- Variational methods
- Davydov Ansatz
- Thermo-Field Dynamic
- Doorway-Window representation

Now:

efficient numerical methods in many-body quantum dynamics and theoretical spectroscopy

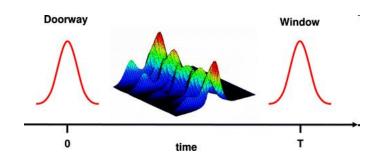
The doorway-window representation Developed by Yan & Mukamel;



Extended by Tanimura, Domcke & Stock, Novoderezkin

W is responsible for time-frequency resolution

T>> pump & probe duration



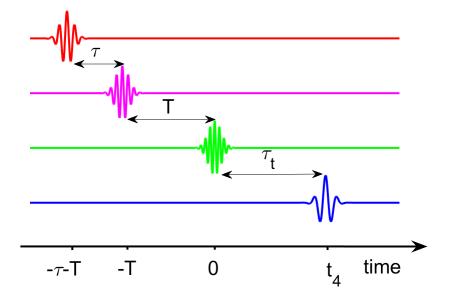
$$S_{\rm pp}(T, \omega_{probe}) = \langle W(\omega_{probe})G(T)D \rangle$$

D is responsible for wavepacket excitation

Frequency of the second (probe) pulse

Time delay between the pulses

Time evolution G(T) is not affected by optical dephasing: We are seating in populations of the system density matrix





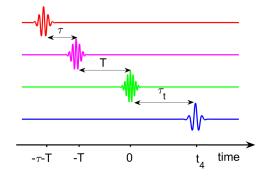
General 4-wave-mixing signal

$$I_{\alpha}(\tau, T, \tau_t) \sim \text{Re} \sum_{k=0, \text{LII}} \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dt_3 \int_{0}^{\infty} dt_2 \int_{0}^{\infty} dt_1 E_1(t + \tau + T - t_3 - t_2 - t_1) \times I_{\alpha}(\tau, T, \tau_t)$$

$$E_2(t+T-t_3-t_2)E_3(t-t_3)E_4(t-\tau_t)e^{i\xi_{\alpha}\omega_{pu}(t_1-\tau)}e^{i\omega_{pr}(t_3-\tau_t)}R_{\alpha k}(t_3,t_2,t_1).$$

DW change of integration variables

 $t = t' + t_3, \ t_2 = t' + T - t_2'$





DW representation of 4-wave-mixing signals

$$I_{\alpha}(\tau, T, \tau_t) \sim \operatorname{Re} \sum_{k=0,I,II} a_k \operatorname{Tr} \left[W_k(\tau_t) \mathcal{L}(T) D_{\alpha}(\tau) \right],$$

$$D_{\alpha}(\tau) = \int_{-\infty}^{\infty} dt_2' \int_{0}^{\infty} dt_1 E_2(t_2') E_1(t_2' + \tau - t_1) e^{i\xi_{\alpha}\omega_{pu}(t_1 - \tau)} \mathcal{D}_{\alpha}(t_1)$$

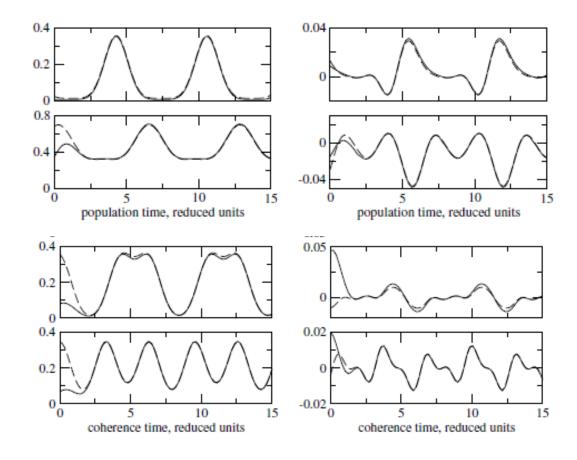
$$W_k(\tau_t) = \int_{-\infty}^{\infty} dt' \int_{0}^{\infty} dt_3 E_4(t' + t_3 - \tau_t) E_3(t') e^{i\omega_p(t_3 - \tau_t)} \mathcal{W}_k(t_3).$$

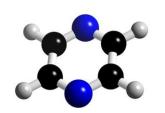


Pump-probe

Figure 1. 2P signal $S_{\rm 2P}(T)$ induced by weak ($\lambda_a=0.01$ eV, (a)) and strong ($\lambda_a=0.1$ eV, (b)) short ($\Gamma_a=\Omega$) pulses. Full black lines, exact calculation; full gray lines, strong-pulse DW approximation. Dotted and dashed lines depict the ground-state and excited-state contributions to the strong-pulse DW signal.

Three-pulse photon echo





HANGILOU DIANZI UM

松州電子科技大学

HANGZHOU DIANZI UNIVERSITY

$$H = \begin{pmatrix} H_0 & 0 & 0 \\ 0 & H_{\rm I} & 0 \\ 0 & 0 & H_{\rm II} \end{pmatrix}$$

$$\mu^{\downarrow} = \begin{pmatrix} 0 & \mu_{0,I} & 0 \\ 0 & 0 & \mu_{I,II} \\ 0 & 0 & 0 \end{pmatrix}, \quad \mu^{\uparrow} = \begin{pmatrix} 0 & 0 & 0 \\ \mu_{I,0} & 0 & 0 \\ 0 & \mu_{II,I} & 0 \end{pmatrix}$$

$\boldsymbol{R}_e(T), \boldsymbol{P}_e(T)$

$$\boldsymbol{R}_g(T), \boldsymbol{P}_g(T)$$

Pyrazine

Energy

manifold {II}

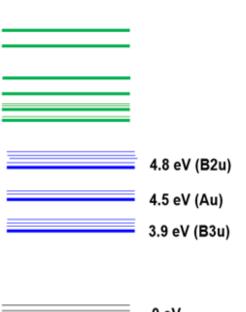
states f_1, f_2, \cdots

manifold {I}

states e_1, e_2, \cdots

manifold {0}

state g





For changing to classical trajectories, make three approximations

I. Classical approximation for electronic coherences:

$$\langle g|e^{iH_0t_3}\boldsymbol{\mu}_{0,\mathrm{I}}e^{-iH_1t_3}|e\rangle \approx e^{iU_{ge}(\boldsymbol{R})t_3}\boldsymbol{\mu}_{ge}(\boldsymbol{R}) \qquad \langle f|e^{iH_{\mathrm{II}}t_3}\boldsymbol{\mu}_{\mathrm{II},\mathrm{I}}e^{-iH_{\mathrm{I}}t_3}|e\rangle \approx e^{iU_{fe}(\boldsymbol{R})t_3}\boldsymbol{\mu}_{fe}(\boldsymbol{R})$$

$$U_{eg}(\boldsymbol{R}) = V_e(\boldsymbol{R}) - V_g(\boldsymbol{R}) \qquad \qquad U_{fe}(\boldsymbol{R}) = V_f(\boldsymbol{R}) - V_e(\boldsymbol{R})$$

II. Classical averaging

Trace over nuclear degrees of freedom is replaced by the sampling of classical trajectories from the classical Wigner distribution

$$ho_B
ightarrow
ho_g^{Wig}(m{R}_g,m{P}_g)_{
m c}$$

Trace over electronic degrees of freedom remains

III. Classical population evolution

T-evolutions of

$$\begin{split} W_{0}(\omega_{pr},\tau_{t},T) &= e^{iH_{0}T}W_{0}(\omega_{pr},\tau_{t})e^{-iH_{0}T}, \\ W_{\mathrm{I}}(\omega_{pr},\tau_{t},T) &= e^{iH_{\mathrm{I}}T}W_{\mathrm{I}}(\omega_{pr},\tau_{t})e^{-iH_{\mathrm{I}}T}, \end{split} \qquad W_{\mathrm{II}}(\omega_{pr},\tau_{t},T) = e^{iH_{\mathrm{I}}T}W_{\mathrm{II}}(\omega_{pr},\tau_{t})e^{-iH_{\mathrm{I}}T}. \end{split}$$

are replaced by evolutions over trajectories in the electronic ground state and lower-lying excited electronic states

Transient absorption pump-probe





$$S_{int}(T, \omega_{pr}) \sim \langle D_0(\omega_{pu}, \mathbf{R}_g, \mathbf{P}_g) W_0^{int}(\omega_{pr}, \mathbf{R}_g(T), \mathbf{P}_g(T)) \rangle +$$

$$\langle D_{\mathrm{I}}(\omega_{pu}, \boldsymbol{R}_{g}, \boldsymbol{P}_{g})W_{\mathrm{I}}^{int}(\omega_{pr}, \boldsymbol{R}_{e}(T), \boldsymbol{P}_{e}(T))\rangle - \langle D_{\mathrm{I}}(\omega_{pu}, \boldsymbol{R}_{g}, \boldsymbol{P}_{g})W_{\mathrm{II}}^{int}(\omega_{pr}, \boldsymbol{R}_{e}(T), \boldsymbol{P}_{e}(T))\rangle.$$

$$D_0(\omega_{pu}, \mathbf{R}_g, \mathbf{P}_g) = \sum_e \mathcal{E}_{pu}^2(\omega_{pu} - U_{eg}(\mathbf{R}_g)) |\mu_{ge}(\mathbf{R}_g)|^2 \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g),$$

$$D_{\mathrm{I}}(\omega_{pu}, \mathbf{R}_g, \mathbf{P}_g) = \mathcal{E}_{pu}^2(\omega_{pu} - U_{eg}(\mathbf{R}_g)) |\mu_{ge}(\mathbf{R}_g)|^2 \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g).$$

$$W_0^{int}(\omega_{pr}, \mathbf{R}_g(T), \mathbf{P}_g(T)) = \sum_e \mathcal{E}_{pr}^2(\omega_{pr} - U_{eg}(\mathbf{R}_g(T))) |\mu_{ge}(\mathbf{R}_g(T))|^2,$$

$$W_{\mathrm{I}}^{int}(\omega_{pr}, \mathbf{R}_e(T), \mathbf{P}_e(T)) = \mathcal{E}_{pr}^2(\omega_{pr} - U_{e(T)g}(\mathbf{R}_e(T))) |\mu_{ge(T)}(\mathbf{R}_e(T))|^2,$$

$$W_{\text{II}}^{int}(\omega_{pr}, \boldsymbol{R}_e(T), \boldsymbol{P}_e(T)) = \sum_{f} \mathcal{E}_{pr}^2(\omega_{pr} - U_{fe(T)}(\boldsymbol{R}_e(T))) |\mu_{e(T)f}(\boldsymbol{R}_e(T))|^2.$$



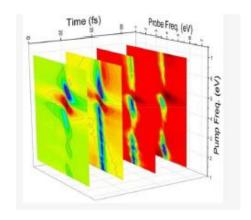
We need just electronic energies and transition dipole moments along trajectories to calculate any nonlinear spectroscopic signal

The numerical effort is comparable to that for the calculation of population evolutions

$$U_{eg}(\mathbf{R}_g(T))$$
 $|\mu_{ge}(\mathbf{R}_g(T))|^2$

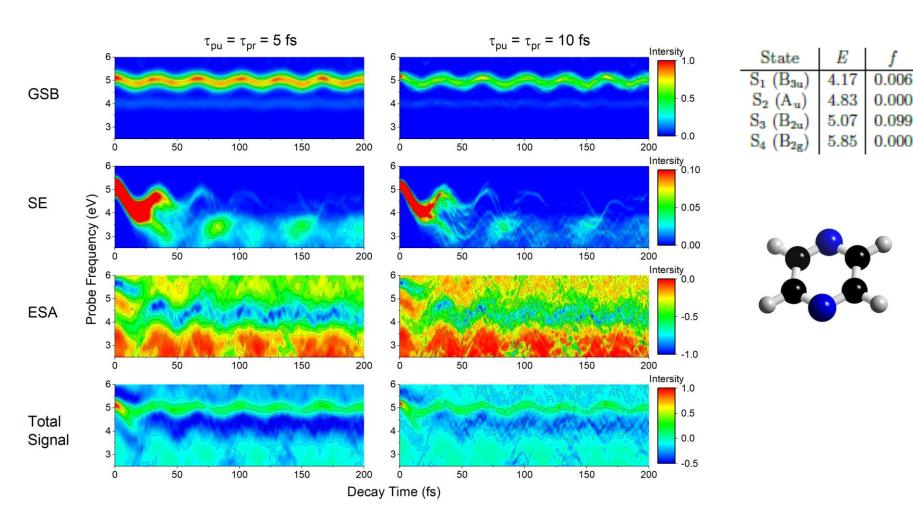
$$U_{e(T)g}(\mathbf{R}_e(T))$$
 $|\mu_{ge(T)}(\mathbf{R}_e(T))|^2$

$$U_{fe(T)}(\boldsymbol{R}_e(T))$$
 $|\mu_{e(T)f}(\boldsymbol{R}_e(T))|^2$



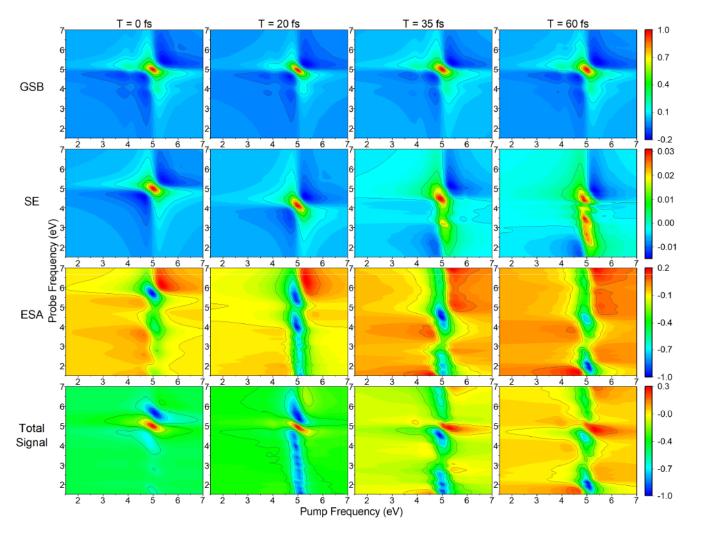


Signals for pyrazine: TA PP integral





Signals for pyrazine: photon-echo 2D



State	\boldsymbol{E}	f
S_1 (B_{3u})	4.17	0.006
$S_2(A_u)$	4.83	0.000
S_3 (B_{2u})	5.07	0.099
S_4 (B_{2g})	5.85	0.000



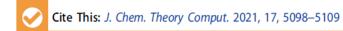


What else?

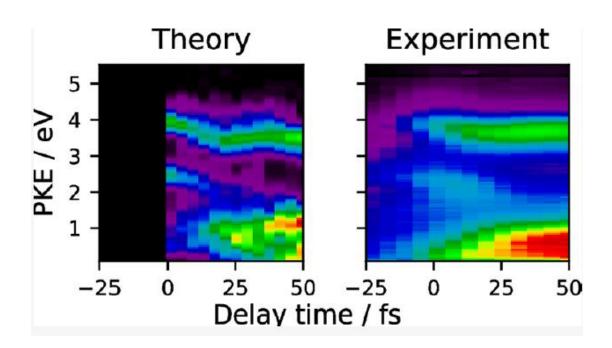


Combined Surface-Hopping, Dyson Orbital, and B-Spline Approach for the Computation of Time-Resolved Photoelectron Spectroscopy Signals: The Internal Conversion in Pyrazine

Tomislav Piteša, Marin Sapunar, Aurora Ponzi, Maxim F. Gelin, Nađa Došlić,* Wolfgang Domcke, and Piero Decleva*







Coupled {0}-{I} manifolds



$$H = \begin{pmatrix} H_0 & H_{0,I} & 0 \\ H_{I,0} & H_I & H_{I,II} \\ 0 & H_{II,I} & H_{II} \end{pmatrix}$$

$$\hat{W}_0^{int} = W_0^{int},$$

$$\hat{W}_I^{int} = \begin{bmatrix} W_I^{int}, & \text{if trajectory stays within } \{I\} \\ -W_0^{int}, & \text{if trajectory jumps from } \{I\} \text{ to } \{0\} \end{bmatrix}$$

$$\begin{bmatrix} W_I^{int} & \text{if trajectory stays within } \{I\} \end{bmatrix}$$

$$\hat{W}_{\text{II}}^{int} = \begin{bmatrix} W_{\text{II}}^{int}, & \text{if trajectory stays within } \{I\} \\ 0, & \text{if trajectory jumps from } \{I\} \text{ to } \{0\} \end{bmatrix}$$

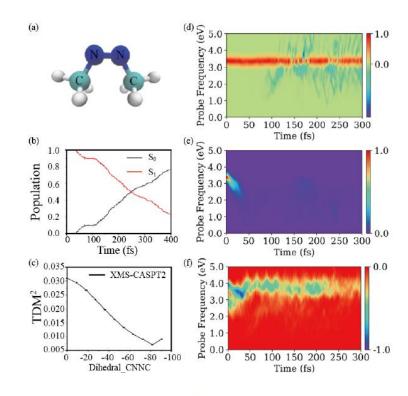
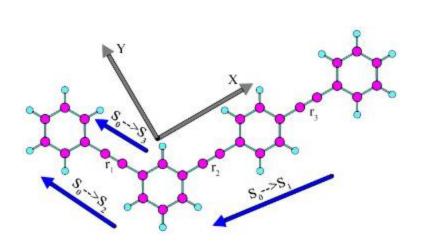


Figure 2: (a) Structure of azomethane. (b) Time-dependent fractional occupations of the S_0 and S_1 electronic states of azomethane in non-adiabatic dynamics staring from the S_1 state. (c) Dependence of the TMD of azomethane on the central torsional angle. Normalized (d) GSB, (e) SE, and (f) ESA contributions to the integral TA PP signal of azomethane as a function of the pump-probe delay time T and the probe-pulse carrier frequency ω_{pr} . Duration of both pump and probe pulses is 5 fs. The carrier frequency ω_{pu} of the pump pulse is tuned into resonance with the $S_1(n\pi^*)$ state. Adapted from ¹⁶³. Copyright American Chemical Society.

Polarization-sensitive signals



$$I_{\alpha}(\tau,T,\tau_t) = \sum_{a,b,c,d=x,y,z} s_p^{(a)} s_p^{(b)} s_u^{(c)} s_u^{(d)} \operatorname{Re} \sum_{k=0,\mathrm{I},\mathrm{II}} a_k \left\langle D_{\alpha}^{(ab)}(\boldsymbol{R},\boldsymbol{P};\tau) W_k^{(cd)}(\boldsymbol{R}(T),\boldsymbol{P}(T);\tau_t) \right\rangle$$



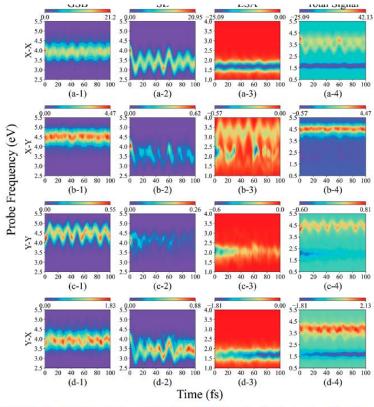


Figure 3. (1) GSB, (2) SE, and (3) ESA contributions and (4) total integral signal $I_{aut}(\tau, \omega_{pr})$ as a function of τ and ω_{pr} . The notation $\alpha - \beta$ ($\alpha, \beta = X, Y$) on the left side indicates that the pump pulse polarization (ε_{pw}) is a long the α -axis direction, while the probe pulse polarization (ε_{pp}) is along the β -axis direction. The central frequency of the pump pulse is $\omega_{pu} = 4.03$ eV, and the pulse durations are $\tau_{pu} = \tau_{pu} = 5$ fs.

Vis pump – Xray probe





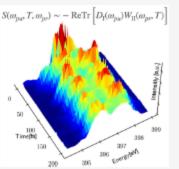
pubs.acs.org/JPCL Letter

Tuning UV Pump X-ray Probe Spectroscopy on the Nitrogen K Edge Reveals the Radiationless Relaxation of Pyrazine: *Ab Initio* Simulations Using the Quasiclassical Doorway—Window Approximation

Tobias Kaczun,* Adrian L. Dempwolff,* Xiang Huang, Maxim F. Gelin, Wolfgang Domcke,* and Andreas Dreuw*



assurablished as a versatile technique for the exploration of ultrafast photoinduced dynamics in valence-excited states. In this work, an ab initio theoretical framework for the simulation of time-resolved UV pump X-ray probe spectra is presented. The method is based on the description of the radiation—matter interaction in the classical doorway—window approximation and a surface-hopping algorithm for the nonadiabatic nuclear excited-state dynamics. Using the second-order algebraic—diagrammatic construction scheme for excited states, UV pump X-ray probe signals were simulated for the carbon and nitrogen K edges of pyrazine, assuming a duration of 5 fs of the UV pump and X-ray probe pulses. It is predicted that spectra measured at the nitrogen K edge carry much richer information about the ultrafast nonadiabatic dynamics in the valence-excited states of pyrazine than those measured at the carbon K edge.



On-the-fly ML-enhanced simulations





Letter



Artificial-Intelligence-Enhanced On-the-Fly Simulation of Nonlinear Time-Resolved Spectra

Sebastian V. Pios, Maxim F. Gelin, Arif Ullah, Pavlo O. Dral,* and Lipeng Chen*

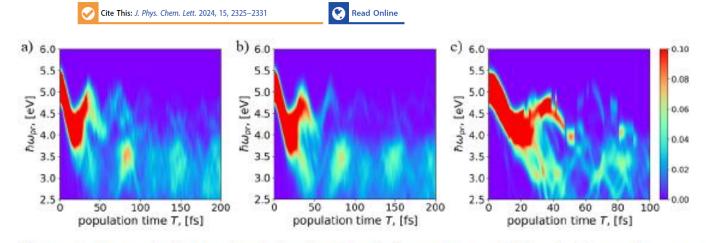


Figure 2: Integral time-resolved stimulated-emission spectrum $I(T, \omega_{pr})$ of pyrazine as a function of the population time T and the carrier frequency ω_{pr} of the probe pulse. (a). The converged spectrum calculated with 600 ML-accelerated trajectories. (b). Reference spectrum calculated with 600 pure ab-initio trajectories. (c). Spectrum calculated from 50 pure ab-initio trajectories (100 fs), which were used for the construction of the ML training set.

Strong-pump strong-probe

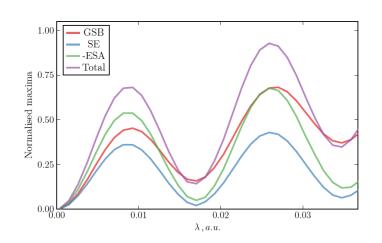


$$D_0(\omega_{pu}, \mathbf{R}_g, \mathbf{P}_g) = \sum_e \frac{\lambda_{pu}^2 |\mu_{ge}(\mathbf{R}_g)|^2}{\Omega_{eg}^2(\mathbf{R}_g)} (1 - \cos(\Omega_{eg}(\mathbf{R}_g)\tau_{pu})) \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g),$$

$$D_{\mathrm{I}}(\omega_{pu}, \mathbf{R}_g, \mathbf{P}_g) = \frac{\lambda_{pu}^2 |\mu_{ge}(\mathbf{R}_g)|^2}{\Omega_{eg}^2(\mathbf{R}_g)} (1 - \cos(\Omega_{eg}(\mathbf{R}_g)\tau_{pu})) \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g)$$

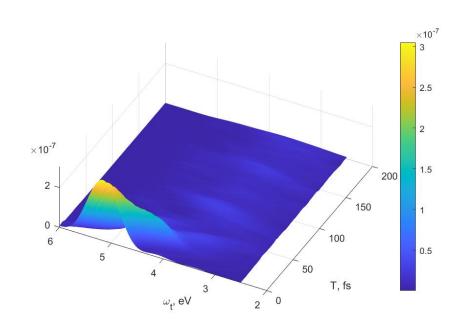
$$\Omega_{eg}(\mathbf{R}_g) = \sqrt{(U_{eg}(\mathbf{R}_g) - \omega_{pu})^2 + 4\lambda_{pu}^2 |\mu_{ge}(\mathbf{R}_g)|^2}$$

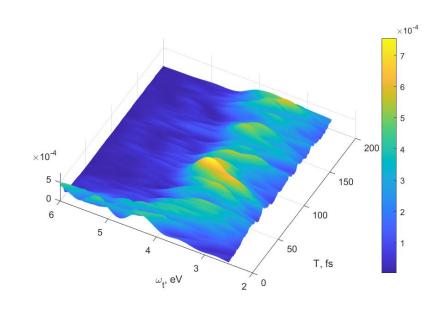




Strong-pump strong-probe



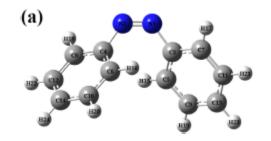




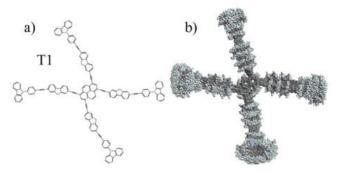


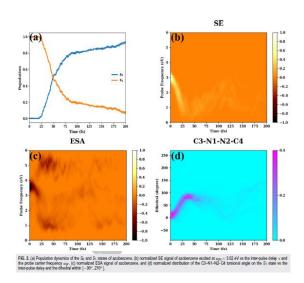


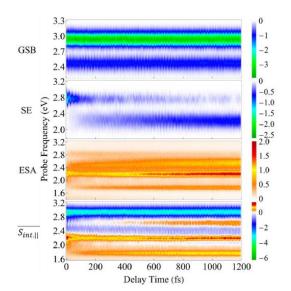
DW combined with mapping approach



DW combined with Ehrenfest dynamics









Plans for the future:

1. New signals

6-wave mixing, UV/Vis pump - IR probe, Odd-wave-mixing at interfaces

2. Better simulation methods NAF (Jian Liu) Account for electronic coherences

3. Comparison with numerically "exact" quantum simulations

Conclusions



The developed simulation method and protocol

- Requires as input electronic energies and dipole moments along classical trajectories in the electronic ground states and lower-laying excited electronic states
- Works for any (quasi)classical trajectory method: Tully's surface hopping, Ehrenfest, mapping approach, etc.
- Fully accounts for pulse effects (shapes, durations, chirps)
- Up to now, has been applied to the simulation of signals of femtosecond UV/vis spectroscopy, photo-electron spectroscopy. and visible pump X-ray probe spectroscopy
- Allows simulation of femtosecond spectroscopic signals of **realistic molecular systems** under **realistic experimental conditions** (strong pulses of any shape) by using **ab initio** methods



Wiley Interdisciplinary Reviews: Computational Molecular Science







Computation of Time-Resolved Nonlinear Electronic Spectra From Classical Trajectories

Maxim F. Gelin¹ | Zhenggang Lan² | Nađa Došlić³ | Wolfgang Domcke⁴ ©

¹School of Science, Hangzhou Dianzi University, Hangzhou, China | ²Key Laboratory of Theoretical Chemistry of Environment, Ministry of Education and Guangdong Provincial Key Laboratory of Chemical Pollution and Environmental Safety, School of Environment, South China Normal University, Guangzhou, China | ³Department of Physical Chemistry, Ruder Bošković Institute, Zagreb, Croatia | ⁴Department of Chemistry, Technical University of Munich, Garching, Germany

Correspondence: Wolfgang Domcke (domcke@ch.tum.de)

Received: 3 October 2024 | Revised: 24 January 2025 | Accepted: 9 February 2025

WaveMixings.jl: a Julia package for performing on-the-fly time-resolved nonlinear electronic spectra from quasi-classical trajectories

Luis Vasquez and Maxim F. Gelin*

School of Science, Hangzhou Dianzi University, Hangzhou 310018,

China

Sebastian Pios and Lipeng Chen

Zhejiang Laboratory, Hangzhou 311100, China

Zhenggang Lan

Key Laboratory of Theoretical Chemistry of Environment,

Ministry of Education and Guangdong Provincial Key Laboratory of

Chemical Pollution and Environmental Safety; School of Environment,

South China Normal University, Guangzhou 510006, P. R. China

Wolfgang Domcke

Department of Chemistry, Technical University of Munich, D-85747 Garching,

Germany

Theory:

Soft: WaveMixings.jl

https://doi.org/10.5281/zenodo.15598121

Collaborators:

Xiang Huang, Wolfgang Domcke TUM, Munich, Germany

Zhenggang Lan SCNU, Guangzhou, China

Weiwei Xie, Nankai University, China

Nadja Došlić Rude Boscovic Institute, Zagreb, Croatia

Lipeng Chen, Sebastian Pios Zhejiang Lab, Hangzhou, China

Pavlo Dral Xiamen University, China

Sebastian Fernandez-Alberti Quilmes University, Argentina

Luis Vasquez Hangzhou Dianzi University,



Funding:

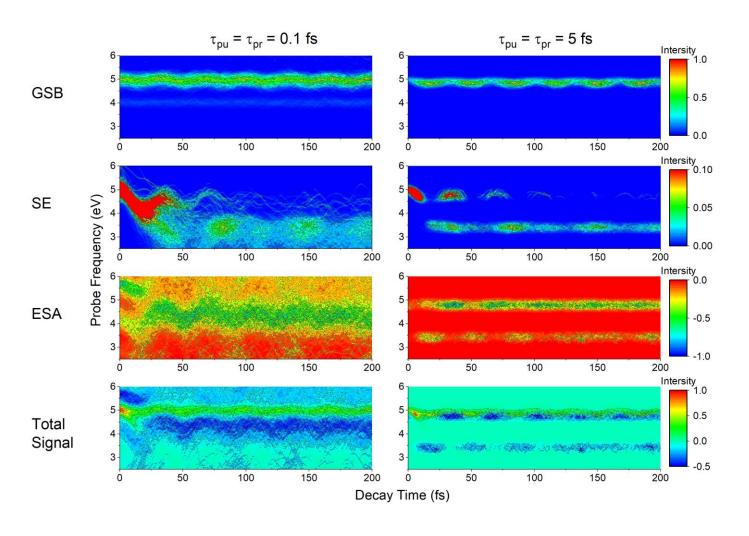
Hangzhou Dianzi University, Startup

NSFC





Signals for pyrazine: TA PP dispersed





2D signal



$$S^{(\alpha)}(\omega_{\tau}, T, \omega_{t}) = S^{(\alpha)}_{GSB}(\omega_{\tau}, T, \omega_{t}) + S^{(\alpha)}_{SE}(\omega_{\tau}, T, \omega_{t}) + S^{(\alpha)}_{ESA}(\omega_{\tau}, T, \omega_{t})$$

$$S_{GSB}^{(\alpha)}(\omega_{\tau}, T, \omega_{t}) \sim \text{Re}\langle D_{0}^{(\alpha)}(\omega_{\tau}, \mathbf{R}_{g}, \mathbf{P}_{g}) W_{0}(\omega_{t}, \mathbf{R}_{g}(T), \mathbf{P}_{g}(T)) \rangle$$

$$S_{SE}^{(\alpha)}(\omega_{\tau}, T, \omega_{t}) \sim \text{Re}\langle D_{\text{I}}^{(\alpha)}(\omega_{\tau}, \boldsymbol{R}_{g}, \boldsymbol{P}_{g}) W_{\text{I}}(\omega_{t}, \boldsymbol{R}_{e(T)}(T), \boldsymbol{P}_{e(T)}(T)) \rangle$$

$$S_{ESA}^{(\alpha)}(\omega_{\tau}, T, \omega_{t}) \sim -\text{Re}\langle D_{I}^{(\alpha)}(\omega_{\tau}, \mathbf{R}_{g}, \mathbf{P}_{g})W_{II}(\omega_{t}, \mathbf{R}_{e(T)}(T), \mathbf{P}_{e(T)}(T))\rangle.$$

$$D_0^{(R)}(\omega_{\tau}, \boldsymbol{R}_g, \boldsymbol{P}_g) = \rho_g^{Wig}(\boldsymbol{R}_g, \boldsymbol{P}_g) \sum_e \frac{\mathcal{E}_1^2(\omega_{\tau} - \omega_p)|\mu_{ge}(\boldsymbol{R}_g)|^2}{\nu - i(U_{eg}(\boldsymbol{R}_g) - \omega_{\tau})},$$

$$D_{\mathrm{I}}^{(R)}(\omega_{\tau}, \mathbf{R}_g, \mathbf{P}_g) = \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g) \frac{\mathcal{E}_1^2(\omega_{\tau} - \omega_p) |\mu_{ge}(\mathbf{R}_g)|^2}{\nu - i(U_{eg}(\mathbf{R}_g) - \omega_{\tau})}$$

$$D_0^{(NR)}(\omega_{\tau}, \boldsymbol{R}_g, \boldsymbol{P}_g) = \rho_g^{Wig}(\boldsymbol{R}_g, \boldsymbol{P}_g) \sum_{e} \frac{\mathcal{E}_1^2(\omega_{\tau} - \omega_p) |\mu_{ge}(\boldsymbol{R}_g)|^2}{\nu + i(U_{eg}(\boldsymbol{R}_g) - \omega_{\tau})},$$

$$D_{\mathrm{I}}^{(NR)}(\omega_{\tau}, \mathbf{R}_g, \mathbf{P}_g) = \rho_g^{Wig}(\mathbf{R}_g, \mathbf{P}_g) \frac{\mathcal{E}_1^2(\omega_{\tau} - \omega_p)|\mu_{ge}(\mathbf{R}_g)|^2}{\nu + i(U_{eg}(\mathbf{R}_g) - \omega_{\tau})}$$

$$W_0(\omega_t, \mathbf{R}_g(T), \mathbf{P}_g(T)) = \sum_e \frac{\mathcal{E}_4^2(\omega_t - \omega_p)|\mu_{ge}(\mathbf{R}_g(T))|^2}{\nu + i(U_{eg}(\mathbf{R}_g(T)) - \omega_t)},$$

$$W_{\mathrm{I}}(\omega_t, \mathbf{R}_{e(T)}(T), \mathbf{P}_{e(T)}(T)) = \frac{\mathcal{E}_4^2(\omega_t - \omega_p)|\mu_{ge(T)}(\mathbf{R}_{e(T)}(T))|^2}{\nu + i(U_{e(T)g}(\mathbf{R}_{e(T)}(T)) - \omega_t)},$$

$$W_{\mathrm{II}}(\omega_t, \mathbf{R}_{e(T)}(T), \mathbf{P}_{e(T)}(T)) = \sum_{f} \frac{\mathcal{E}_4^2(\omega_t - \omega_p) |\mu_{e(T)f}(\mathbf{R}_{e(T)}(T))|^2}{\nu + i(U_{fe(T)}(\mathbf{R}_{e(T)}(T)) - \omega_t)}$$



(ii) On-the-fly simulations of nonlinear femtosecond spectroscopic signals



Read Online

Cite This: J. Phys. Chem. Lett. 2023, 14, 5648-5656

