Vista Seminar
May 7th, 2025

Quantum Trajectories: Discrete or Continuous?

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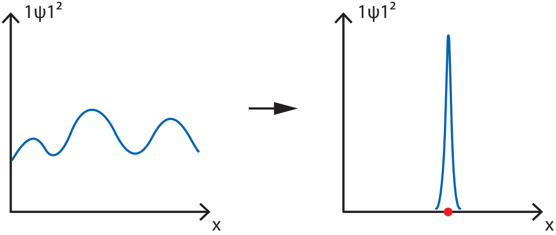
Overview of Trajectory Theories

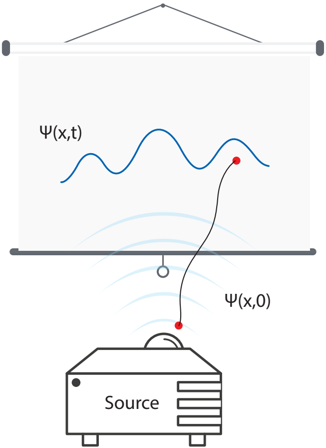
Problems with Standard QM



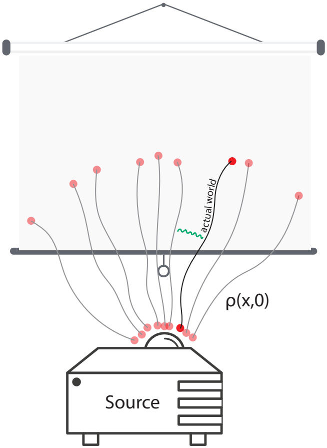
$$\Psi\left(x,t\right) = C_1\psi_1 + C_2\psi_2 + \ldots \xrightarrow{Collapse\ \ evolution} \psi_i$$

Trajectory Approaches: Overview

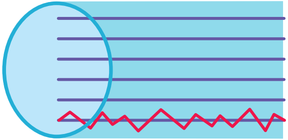
Pilot Wave (PW) Theory



Interacting Quantum Trajectories (IQT) Theory (✓)



The Discrete Trajectory Approach in Detail



IQT Theory (Discrete)

Ontology: Particles + many trajectories (discretely distributed).

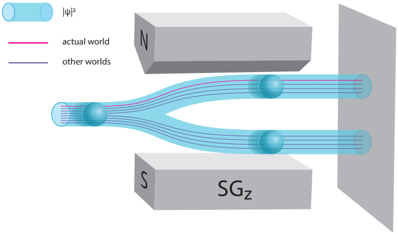
Macroscopic Dynamics:

$$m\ddot{x} = -\frac{\partial V(x)}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left[\frac{\frac{\partial^2}{\partial x^2}\sqrt{\rho}}{\sqrt{\rho}}\right]$$

Microscopic Dynamics: ???

Predictions: Agrees with QM approximately.

Strengths of Discrete IQT Theory: Probability



Dynamics:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}\left(-i\hbar\nabla - e\vec{A}\right)^2 + eV\right]\psi - \mu_B\vec{\sigma}\cdot\vec{B}\psi$$

$$\psi(\vec{x},0) = \phi(x,y,z)\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}\right) \xrightarrow{\text{time evolution}} \frac{1}{\sqrt{2}}\begin{bmatrix}\phi_\uparrow(x,y,z,t)\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\\phi_\downarrow(x,y,z,t)\end{bmatrix}$$

Probability[1]**:**

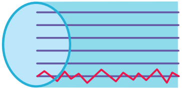
$$\Pr(\uparrow) = \frac{\#\text{ of trajectories that go up}}{\#\text{ of trajectories}}, \qquad \Pr(\downarrow) = \frac{\#\text{ of trajectories that go down}}{\#\text{ of trajectories}}$$

[1] C. Sebens, "Quantum Mechanics as Classical Physics", *Philosophy of Science*, 82, 266-291.

Strengths of Discrete IQT Theory: New Physics

Extracting deeper physics from:

$$m\ddot{x} = -\frac{\partial V(x)}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left[\frac{\frac{\partial^2}{\partial x^2}\sqrt{\rho}}{\sqrt{\rho}}\right]$$



Toy Model[2]: N-trajectory, 1-D model

$$m\ddot{x}_i = -\frac{\partial V(x_i)}{\partial x_i} + F_{Q_i},$$

where

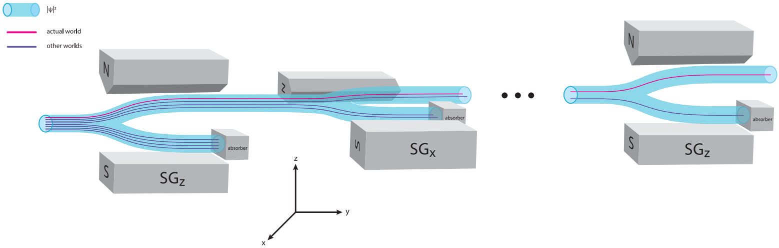
$$F_{Q_i} = \frac{\hbar^2}{4m}\left[\frac{1}{(x_{i+1}-x_i)^2}\left(\frac{1}{x_{i+2}-x_{i+1}} - \frac{2}{x_{i+1}-x_i} - \frac{1}{x_i - x_{i-1}}\right)\right.$$
$$\left. - \frac{1}{(x_i - x_{i-1})^2}\left(\frac{1}{x_{i+1}-x_i} - \frac{2}{x_i - x_{i-1}} + \frac{1}{x_{i-1}-x_{i-2}}\right)\right]$$

$x_0 = -\infty$, $x_{N+1} = \infty$.

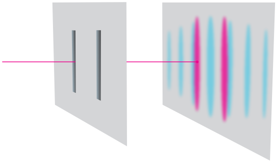
[2] M. J. Hall, D.-A. Deckert, and H. J. Wiseman, "Quantum Phenomena Modeled by Interactions between Many Classical Worlds", *Phys. Rev.* X 4, 041013.

The Problem of Sparse Trajectories

Discrete IQT Theory and Repeated Measurements



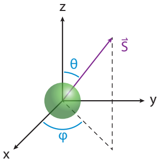
Does splitting continue to occur when the trajectories are sparse?



Modeling Sparse Trajectories in an SG Setup

Model:

$$m\ddot{z}_i = -\frac{\partial V(z_i)}{\partial z_i} + F_{Q_i} + F_{Qs_i} + \mu_B B_0' \cos\theta,$$



where

$$\mathrm{F}_{Q_i} = \frac{\hbar^2}{4m}\left[\frac{1}{(z_{i+1}-z_i)^2}\left(\frac{1}{z_{i+2}-z_{i+1}} - \frac{2}{z_{i+1}-z_i} - \frac{1}{z_i-z_{i-1}}\right) - \frac{1}{(z_i-z_{i-1})^2}\left(\frac{1}{z_{i+1}-z_i} - \frac{2}{z_i-z_{i-1}} + \frac{1}{z_{i-1}-z_{i-2}}\right)\right],$$

$$F_{Qs_i} = \frac{\hbar^2}{4m}\left[\frac{(\phi_i-\phi_{i-1})^2\sin^2\left(\frac{\theta_i+\theta_{i-1}}{2}\right) + (\theta_i-\theta_{i-1})^2}{(z_i - z_i)^3} - \frac{(\phi_{i+1}-\phi_i)^2\sin^2\left(\frac{\theta_{i+1}+\theta_i}{2}\right) + (\theta_{i+1}-\theta_i)^2}{(z_{i+1}-z_i)^3}\right],$$

$$\dot{\theta} = \frac{\hbar}{2m\sin\theta_i}\left[\frac{(\phi_{i+1}-\phi_i)\sin\left(\frac{\theta_{i+1}+\theta_i}{2}\right)}{(z_{i+1}-z_i)^2} - \frac{(\phi_i-\phi_{i-1})\sin\left(\frac{\theta_i+\theta_{i-1}}{2}\right)}{(z_i-z_{i-1})^2}\right],$$

$$\dot{\phi} = \frac{\hbar}{2m\sin\theta_i}\left[\left(\frac{\phi_{i+1}-\phi_{i-1}}{z_{i+1}-z_{i-1}}\right)^2 \sin\theta_i \cos_i - \frac{\theta_{i+1}-\theta_i}{(z_{i+1}-z_i)^2} + \frac{\theta_i-\theta_{i-1}}{(z_i-z_{i-1})^2}\right] + \frac{2}{\hbar}\mu_B B_0' \cos\theta_i z_i,$$

where[3] $z_0 = -\infty, z_{N+1} = \infty$.

[3] R. Lombardini and B. Poirier, "Interacting Quantum Trajectories for Particles with Spin 1/2, *Mol. Phys. 2024, e2334805.*

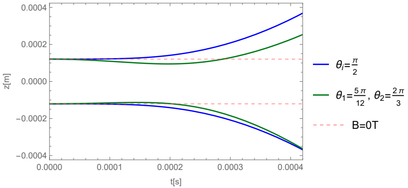
Low N Trajectories in a SG Setup

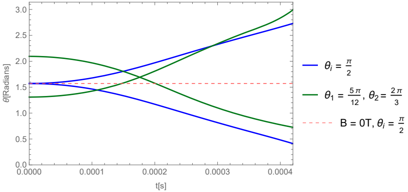
For N=2, $B_0' = 100$**T/m:**

$$\ddot{z}_1 = \frac{\hbar^2}{2m^2(z_1 - z_2)^3}\left[1 - \frac{1}{2}(\phi_1 - \phi_2)^2 \sin^2\left(\frac{\theta_2 + \theta_1}{2}\right) - \frac{1}{2}(\theta_2 + \theta_1)^2\right] + \frac{\mu_B}{m} B_0' \cos\theta_1,$$

$$\dot{\theta}_1 = \frac{\hbar}{2m}\left[\frac{(\phi_2 - \phi_1)\sin^2\left(\frac{\theta_2 + \theta_1}{2}\right) + (\theta_2 - \theta_1)^2}{\sin\theta_1 (z_1 - z_2)^2}\right],$$

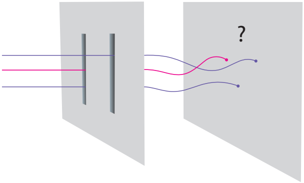
$$\dot{\phi}_1 = \frac{\hbar}{2m\sin\theta_1}\frac{\theta_1 - \theta_2}{(z_1 - z_2)^2} + \frac{2}{\hbar}\mu_B B_0' z_1.$$





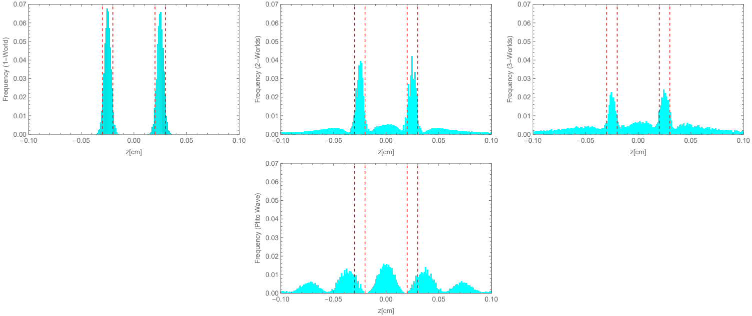
Low N Trajectories in a Double-Slit Setup

For N=2, $w = .01$**mm,** $d = .05$**mm:**



$$\ddot{z}_1 = \frac{\hbar^2}{2m^2(z_1 - z_2)^3}$$

$$\ddot{z}_2 = \frac{\hbar^2}{2m^2(z_2 - z_1)^3}$$



Possible Responses

Responses to the Sparse Trajectory Problem

- **Response 1:** There is no problem. The theory is just making testable predictions.
- **Response 2:** Maybe more realistic dynamics can still produce results that agree with QM.
- **Response 3:** Maybe the density of trajectories is just very high.

Recovering QM From More Realistic Physics

What went wrong with the toy model?

$$m\ddot{z}_i = -\frac{\partial V(z_i)}{\partial z_i} + F_{Q_i} + F_{Qs\,i} + \mu_B B_0' \cos\theta,$$

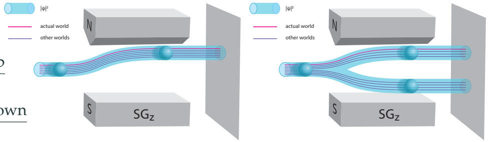
where

$$F_{Q_i} = \frac{\hbar^2}{4m}\left[\frac{1}{(z_{i+1}-z_i)^2}\left(\frac{1}{z_{i+2}-z_{i+1}} - \frac{2}{z_{i+1}-z_i} - \frac{1}{z_i - z_{i-1}}\right)\right.$$
$$\left. - \frac{1}{(z_i - z_{i-1})^2}\left(\frac{1}{z_{i+1}-z_i} - \frac{2}{z_i - z_{i-1}} + \frac{1}{z_{i-1}-z_{i-2}}\right)\right]$$

Splitting + Configuration Space Locality

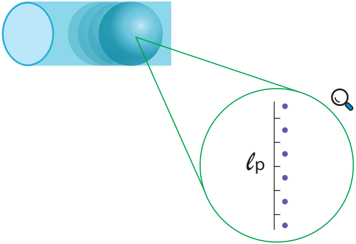
$$\Pr(\uparrow) = \frac{\text{\# of trajectories that go up}}{\text{\# of trajectories}}$$

$$\Pr(\downarrow) = \frac{\text{\# of trajectories that go down}}{\text{\# of trajectories}}$$



The Immense Density Response

Suppose we have one trajectory per Planck length



$$l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6255 \times 10^{-35}\text{m}$$

along the length of the incident wave-packet.

Then, roughly 106 splittings would thin the initial trajectory density down to one trajectory.

If no amount of splitting will dilute the density enough to see results that deviate from QM, then discrete and continuous IQT theory are indistinguishable.

Thank you

Questions?

Appendix

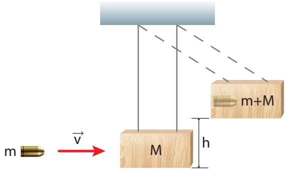
Solutions to the Problems

- **Stochastic collapse approaches** - Quantum dynamics are modified to include a stochastic element that induces collapses.

- **Many worlds approaches** - No collapse occurs. Rather, the wavefunction branches during measurement into many worlds.

- **Trajectory approaches** - The world is populated by particles that follow well-defined trajectories. (✓)

Measurement in Classical Physics



Using conservation laws:

$$v = \frac{M+m}{m}\sqrt{2gh}$$

- Measurement process is completely described by the dynamics.
- A notion of measurement is not fundamental to the theory.
- The theory's dynamics informs us of what a potential measurement is.