



國立交通大學  
National Chiao Tung University

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**Global switching trajectory surface hopping molecular dynamics simulation on on-the-fly TDDFT potential energy surfaces**

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**Virtual International Seminar on  
Theoretical Advancements**





# Outline

- Global switching algorithm
- Local switching via global switching
- TDDFT potential energy surfaces
  1. dMe-OMe-NAIP photoisomerization (41 atoms)
  2. Retinal chromophore isomer photoisomerization (63 atoms)
- Conclusions



# Global switching algorithm



## Mixed quantum-classical electronic coupled equations

$$i\hbar \frac{\partial \Psi(\mathbf{r}, \mathbf{R}, t)}{\partial t} = (T_N + H_e(\mathbf{r}, \mathbf{R}))\Psi(\mathbf{r}, \mathbf{R}, t)$$

**$\mathbf{R}$  is from classical trajectory**

**Nuclear kinetic operator**

$$T_N = - \sum_{\alpha=1} \frac{\hbar^2}{2M_\alpha} \nabla_{\mathbf{R}_\alpha}^2 = 0$$

**Electronic basis expansion**

$$\Psi(\mathbf{r}, \mathbf{R}, t) = \sum_j c_j(\mathbf{R}, t) \Psi_j^{\text{BO}}(\mathbf{r}, \mathbf{R})$$

**Electronic time-dependent Schrödinger equations**

$$i\hbar \dot{c}_k(t) = \sum_j c_j(t) \left[ U_{kj}(\mathbf{R}) - i\hbar \dot{\mathbf{R}} \cdot \mathbf{d}_{kj} \right]$$

$$P_{2 \rightarrow 1}(t) = \max \left( 0, - \frac{\dot{\rho}_{22}(t) dt}{\rho_{22}(t)} \right)$$

**Local switching (Tully fewest)**



## Mixed quantum-classical dynamics (**Laudau-Zener model**)

$$T_N = 0$$

$$\begin{pmatrix} V_{11}(\mathbf{R}) & V_{12}(\mathbf{R}) \\ V_{21}(\mathbf{R}) & V_{22}(\mathbf{R}) \end{pmatrix} \begin{bmatrix} c_1(\mathbf{R}) \\ c_2(\mathbf{R}) \end{bmatrix} = i\hbar \frac{\partial}{\partial t} \begin{bmatrix} c_1(\mathbf{R}) \\ c_2(\mathbf{R}) \end{bmatrix}$$

### Linear crossing model

$$V_{11}(x) = -F_1 x \quad V_{22}(x) = -F_2 x \quad V_{12}(x) = V_{21}(x) = V_0$$

Classical trajectory

$$x = vt$$

Scattering matrix

$$\begin{pmatrix} c_1(\infty) \\ c_2(\infty) \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} c_1(-\infty) \\ c_2(-\infty) \end{pmatrix}$$



$$S^R(dia) = \begin{pmatrix} \sqrt{p} & -\sqrt{1-p}e^{i\Phi} \\ \sqrt{1-p}e^{-i\Phi} & \sqrt{p} \end{pmatrix}$$

$$p = e^{-2\delta}$$

$$\delta = \frac{\pi}{8\sqrt{a^2b^2}}$$

This is an exact solution under  $T_N = 0$

Adiabatic switching probability



## Nonadiabatic switching probability expressed in terms of

### Two unitless parameters

$$a^2 = \frac{\hbar^2}{2\mu} \frac{\sqrt{|F_1 F_2|} (|F_2 - F_1|)}{8V_0^3}$$

→ **Effective coupling**

$$b^2 = (E_{//} - E_0) \frac{(|F_2 - F_1|)}{2\sqrt{|F_1 F_2|} V_0}$$

→ **Effective collision energy**

M. S. Child, *Molecular Collision Theory* (Academic, London, New York, 1974)



## Semiclassical solution in time-independent framework

### Coupled Schrödinger equation




$$T_N \neq 0 \quad \left( -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \begin{bmatrix} V_{11}(R) & V_{12}(R) \\ V_{12}(R) & V_{22}(R) \end{bmatrix} \right) \begin{bmatrix} c_1(R) \\ c_2(R) \end{bmatrix} = E \begin{bmatrix} c_1(R) \\ c_2(R) \end{bmatrix}$$

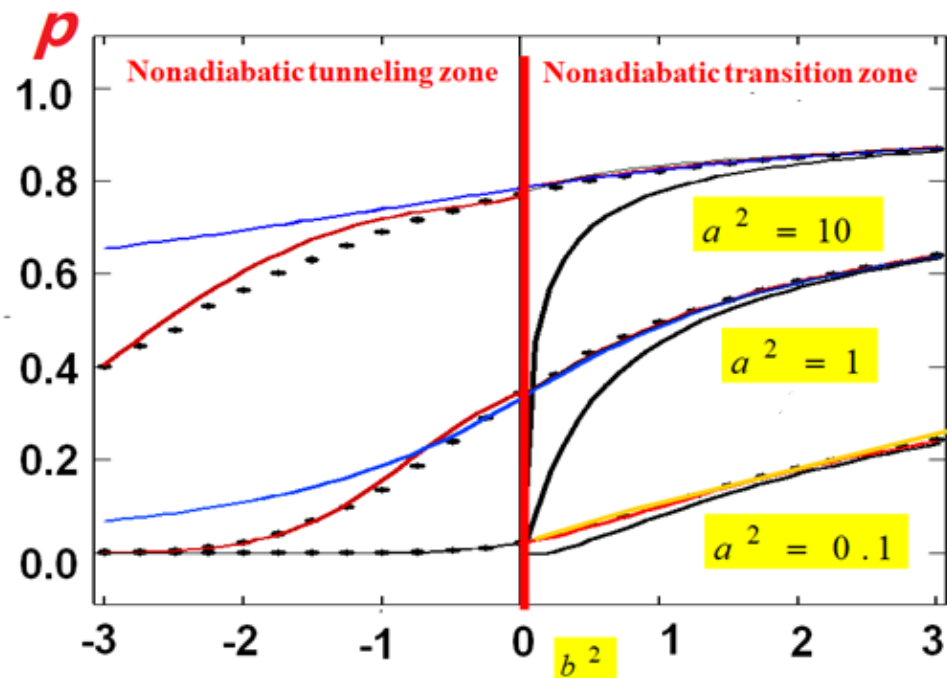
$$p = \exp \left[ -\frac{\pi}{4a} \left( \frac{2}{b^2 + \sqrt{b^4 \pm 1}} \right)^{1/2} \right] \xrightarrow{b^2 \gg 1} p = \exp \left[ -\frac{\pi}{4ab} \right]$$

**Zhu-Nakamura (work at Ex)**      **Landau-Zener (not work at Ex)**

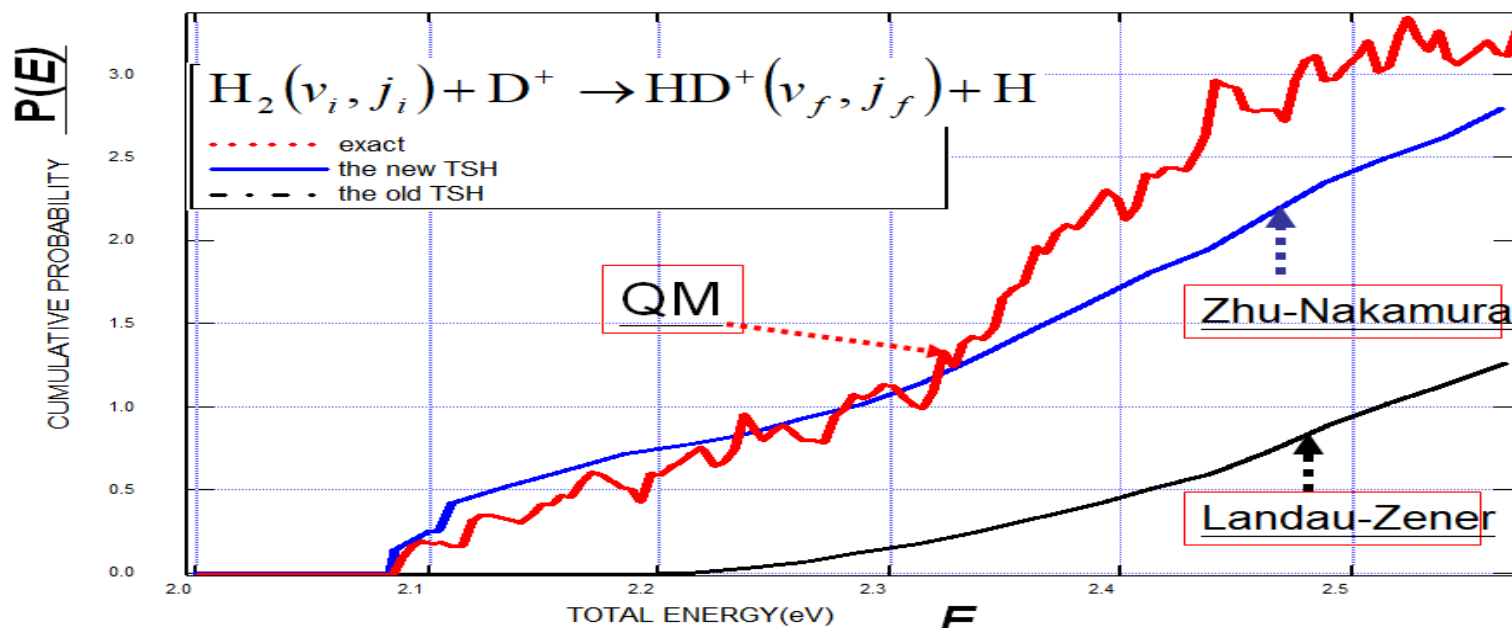
C. Zhu and H. Nakamura, J. Chem. Phys. 101, 10630 (1994); 102, 7448(1995)



Exact   
 Zhu-Nakamura   
 Landau-Zener 



### Reactive charge transfer





Re-expressed in terms of Cartesian coordinates

$\mathbf{R}_1 \quad \mathbf{R}_2$

For diatom

$$\frac{|F_2 - F_1|}{\sqrt{\mu}} = \sqrt{\sum_{i=1}^{N=2} \frac{1}{m_i} \sum_{\alpha=x,y,z} (F_2^{i\alpha} - F_1^{i\alpha})^2}$$

$$\frac{\sqrt{|F_2 F_1|}}{\sqrt{\mu}} = \sqrt{\sum_{i=1}^{N=2} \frac{1}{m_i} \sum_{\alpha=x,y,z} F_2^{i\alpha} F_1^{i\alpha}}$$

$$a^2 = \frac{\hbar^2}{2\mu} \frac{\sqrt{|F_1 F_2|} (|F_2 - F_1|)}{8V_0^3}$$

$$b^2 = (E_{//} - E_0) \frac{|F_2 - F_1|}{2\sqrt{|F_1 F_2|} V_0}$$

$$F_1^{i\alpha} = -\frac{\partial V_1}{\partial R_{i\alpha}}$$

$$F_2^{i\alpha} = -\frac{\partial V_2}{\partial R_{i\alpha}}$$

Directly extending to multi-dimensional for N atoms

$2 \rightarrow N$

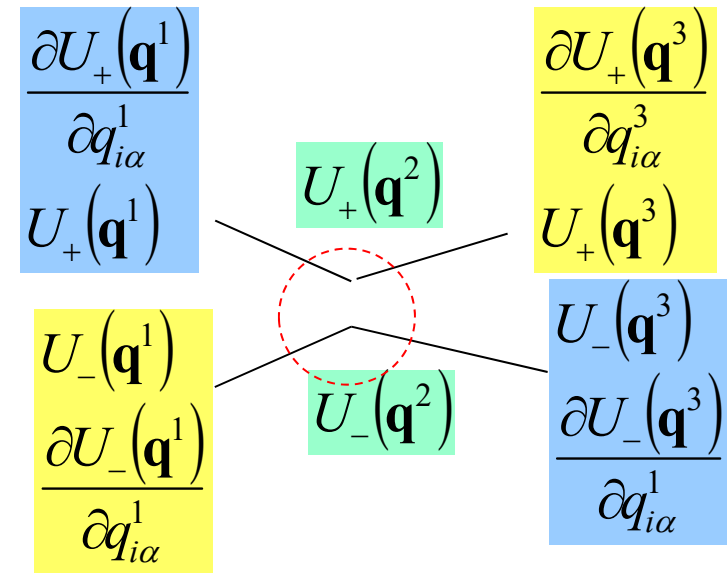


## Multi-dimensional forces along a trajectory at avoided crossing

Linear connection for each i-component

$$R \rightarrow \mathbf{q}$$

$$\mathbf{q} \equiv (q_1, q_2, \dots, q_N)$$



$$F_1^{i\alpha}(\mathbf{q}) = \frac{1}{q_{i\alpha}^3 - q_{i\alpha}^1} \left[ \frac{\partial U_-}{\partial q_{i\alpha}^3} (q_{i\alpha} - q_{i\alpha}^1) - \frac{\partial U_+}{\partial q_{i\alpha}^1} (q_{i\alpha} - q_{i\alpha}^3) \right]$$

$$F_2^{i\alpha}(\mathbf{q}) = \frac{1}{q_{i\alpha}^3 - q_{i\alpha}^1} \left[ \frac{\partial U_+}{\partial q_{i\alpha}^3} (q_{i\alpha} - q_{i\alpha}^1) - \frac{\partial U_-}{\partial q_{i\alpha}^1} (q_{i\alpha} - q_{i\alpha}^3) \right]$$

Linear diabaticization along a trajectory at avoided crossing



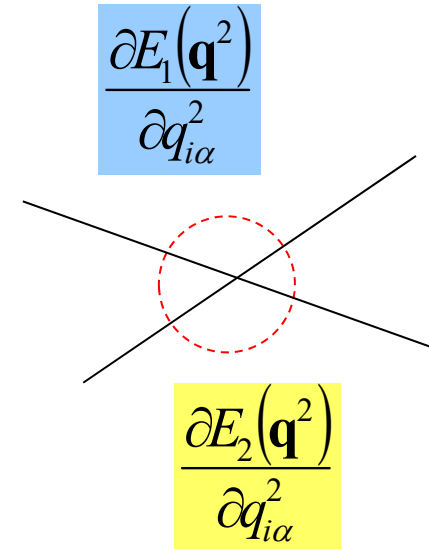
## Between different spin states

Linear connection for each i-component

$$\mathbf{q} \equiv (q_1, q_2, \dots, q_N)$$

$$F_1^{i\alpha}(\mathbf{q}) = \frac{\partial E_1(\mathbf{q}^2)}{\partial q_{i\alpha}^2}$$

$$F_2^{i\alpha}(\mathbf{q}) = \frac{\partial E_2(\mathbf{q}^2)}{\partial q_{i\alpha}^2}$$



Directly given for different spin states



## Define hopping direction

$$s_{i\alpha} = \left[ F_2^{i\alpha}(\mathbf{q}^2) - F_1^{i\alpha}(\mathbf{q}^2) \right] \frac{1}{\sqrt{m_i}}$$

## Normalization

$$\sum_{\alpha=1}^N \dot{\mathbf{R}}_{\alpha} \cdot \mathbf{d}_{12}^{\alpha} \leftarrow \mathbf{n}_i$$

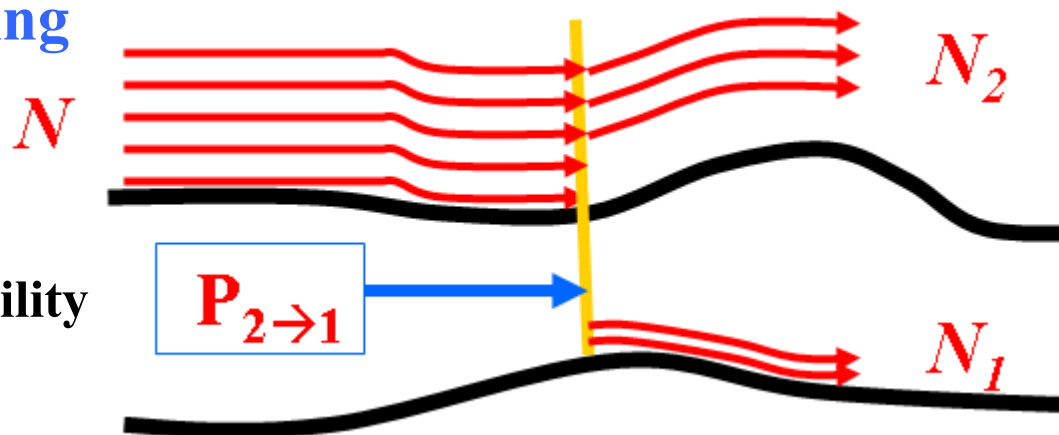
$$\mathbf{n}_i = \frac{1}{\sqrt{s_{ix}^2 + s_{iy}^2 + s_{iz}^2}} (s_{ix}, s_{iy}, s_{iz})$$

$$\mathbf{n}_i^2 = 1$$

Provide the maximum switching probability at avoided crossing

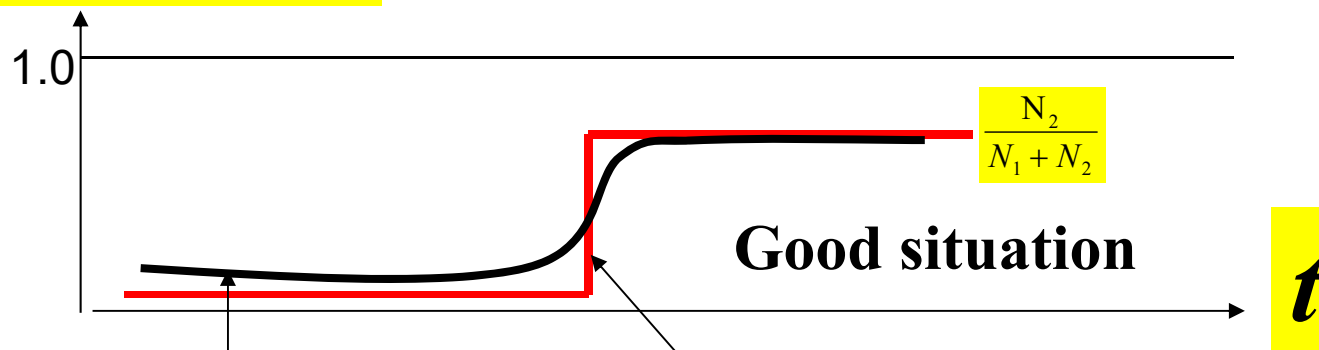


## Trajectory surface hopping



Nonadiabatic switching probability

## Population distribution



$$P_{2 \rightarrow 1}(t) = \max \left( 0, -\frac{\dot{\rho}_{22}(t) dt}{\rho_{22}(t)} \right)$$

Local switching (Tully fewest)

$$T_N = 0$$

$$p_{ZN} = \exp \left[ -\frac{\pi}{4a} \left( \frac{2}{b^2 + \sqrt{b^4 \pm 1}} \right)^{1/2} \right]$$

Global switching (ZN formula)

$$T_N \neq 0$$



## Global nonadiabatic switching algorithm

Only need

Potential energy surfaces

Gradient of potential energy surfaces

$$U_+(\mathbf{R}) = U_+(q_1, q_2, \dots, q_{3N})$$
$$U_-(\mathbf{R}) = U_-(q_1, q_2, \dots, q_{3N})$$

$$\left( \frac{\partial U_+(\mathbf{q})}{\partial q_i} \right)$$

$$\left( \frac{\partial U_-(\mathbf{q})}{\partial q_i} \right)$$

## Run nonadiabatic molecular dynamics simulation

### Searching conical intersections

L. Yu, C. Xu, Y. Lei, C. Zhu, and Z. Wen, PCCP 16, 25883 (2014)

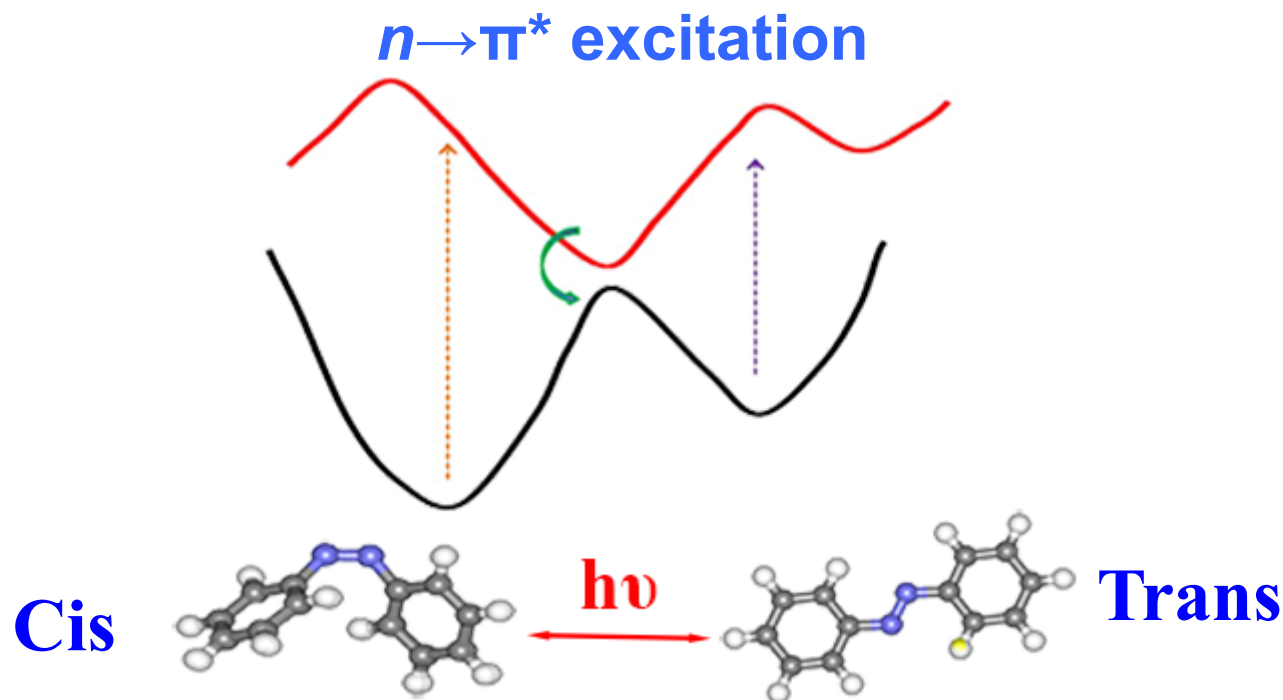


# Local switching via global switching





# Azobenzene photoisomerization



Cis-to-trans and trans-to-cis



**Exactly same initial conditions and the same**  
**Potential energy surfaces: OM2/MR-CISD(10,10)**

**Sampling trajectories:**

**Starting from cis-azobenzene(800)**

**Starting from trans-azobenzene(600),**

**Time step for trajectory propagation is 0.1fs.**

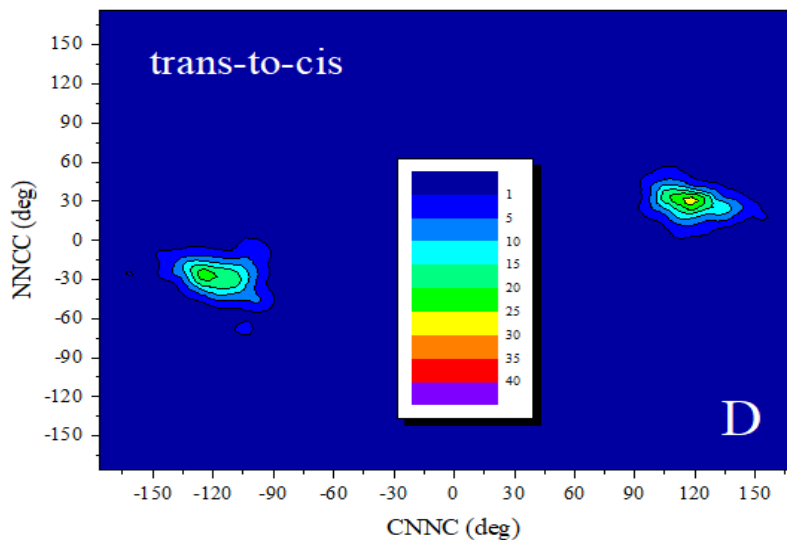
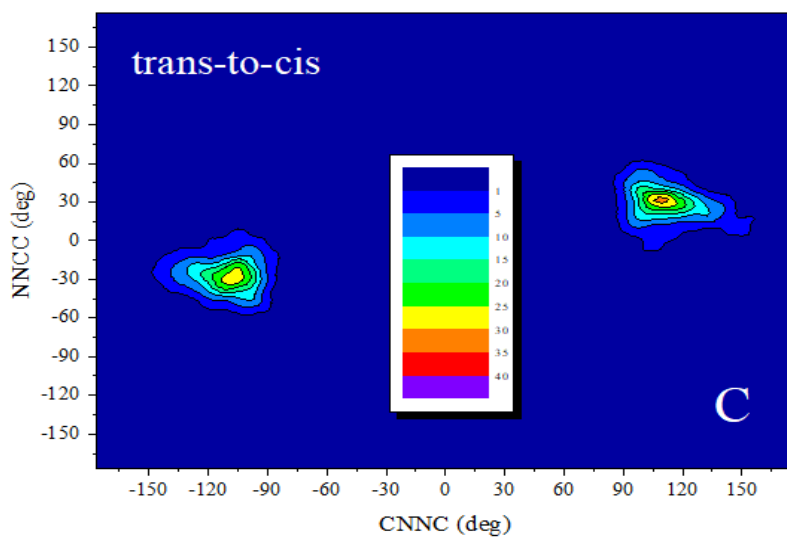
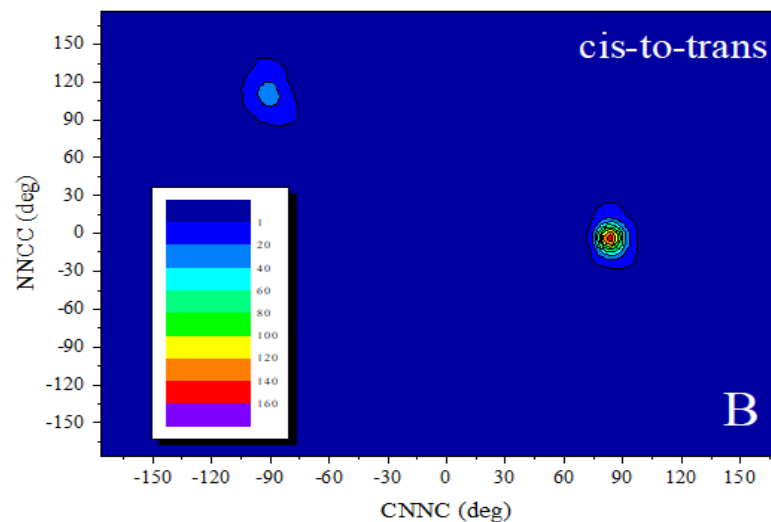
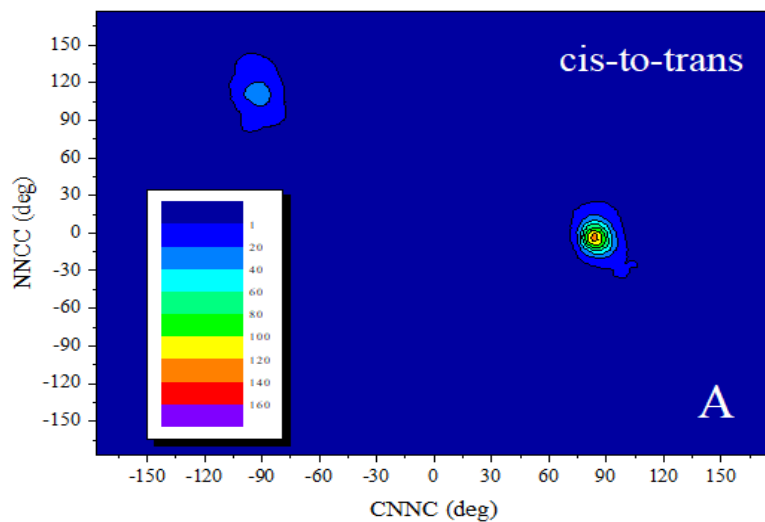
	Quantum yield	lifetimes
<b>Cis-to-trans</b>	<b>Global: 0.57</b> <b>Local : 0.58</b>	<b>Global: 72fs</b> <b>Local : 82fs</b>
<b>Trans-to-cis</b>	<b>Global: 0.16</b> <b>Local : 0.20</b>	<b>Global: 280fs</b> <b>Local : 310fs</b>



# Hopping spot distributions in terms of CNNC dihedral angle

**Local**

**Global**

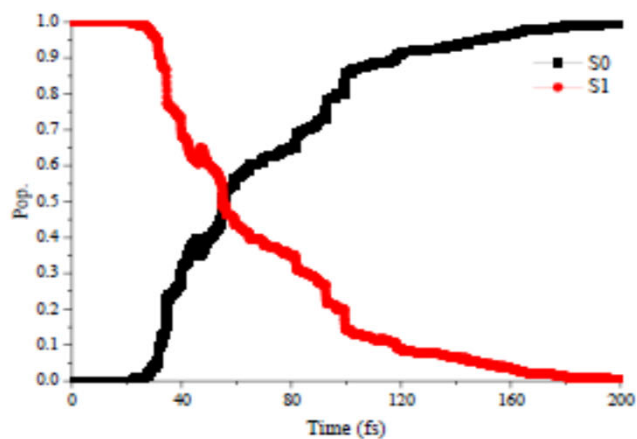




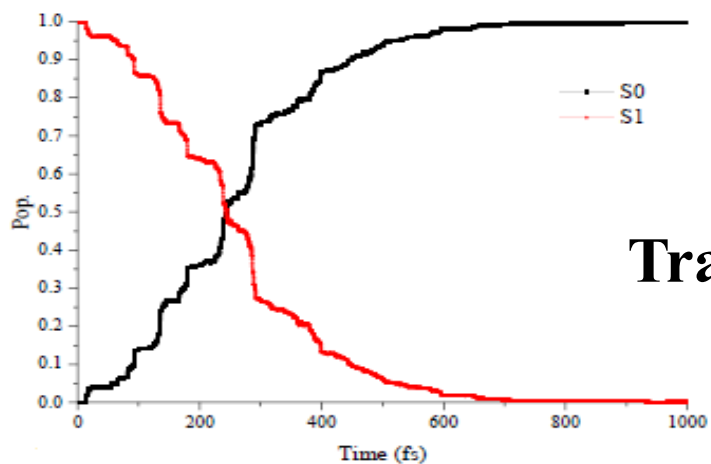
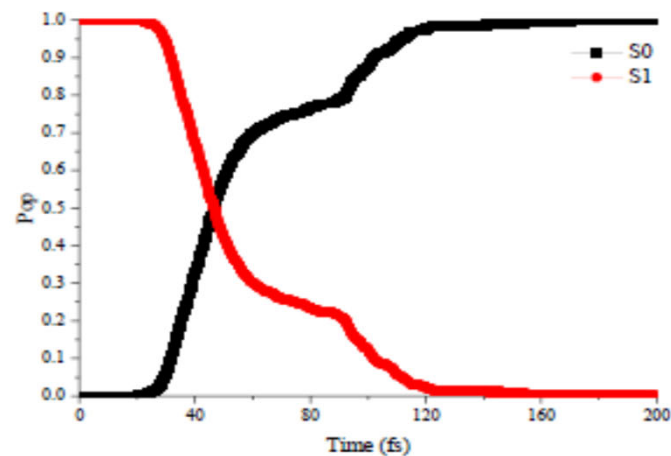
# Average population distributions as function of time

**Local**

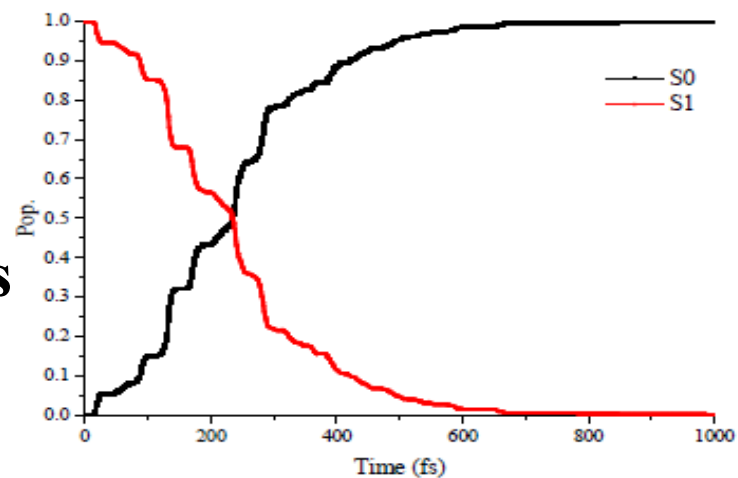
**Global**



**Cis-to-trans**



**Trans-to-cis**





## Local switching (Tully fewest)

## Global switching

$$P_{2 \rightarrow 1}(t) = \max \left( 0, -\frac{\dot{\rho}_{22}(t)dt}{\rho_{22}(t)} \right)$$

$$p_{ZN} = \exp \left[ -\frac{\pi}{4a} \left( \frac{2}{b^2 + \sqrt{b^4 \pm 1}} \right)^{1/2} \right]$$

**Equally good**

L. Yue, L. Yu, C. Xu, Y. Lei, Y. Liu, C. Zhu, ChemPhysChem 18,1274 (2017)

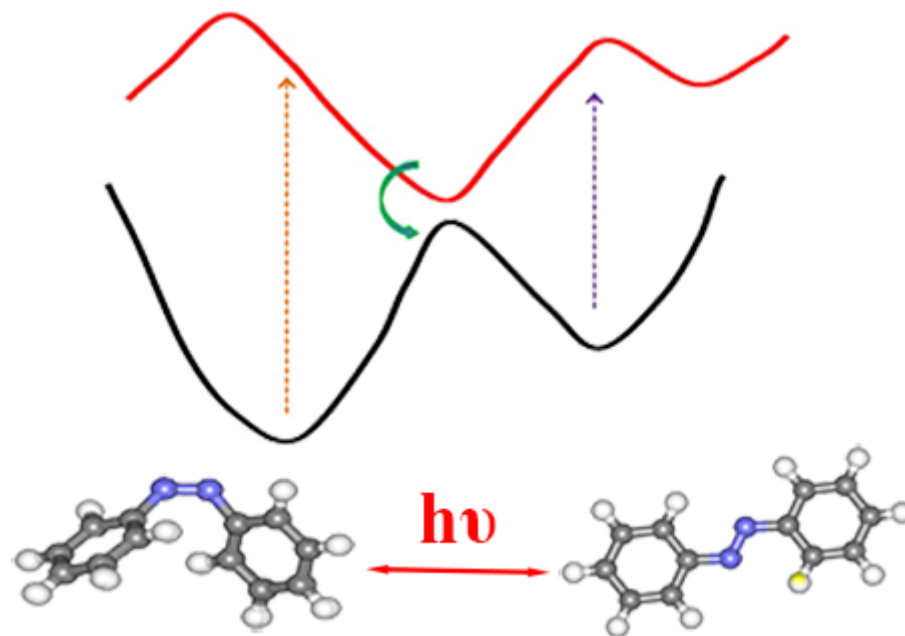


# TDDFT potential energy surfaces



# Azobenzene photoisomerization

$n \rightarrow \pi^*$  excitation



Cis-to-trans and trans-to-cis



## Comparison among SF-TDDFT, TDDFT, CASSCF

Method	N <sub>traj</sub> (cis/trans)	cis- to-trans		trans-to-cis	
		QY	Lifetime(fs)	QY	Lifetime(ps)
SF-TDDFT	300/226	0.43±0.07	63.1±1.1	0.11~0.16	2.218±0.010
LR-TDDFT	259/268	0.34±0.09	62.0±0.9	0.13±0.16	1.039±0.009
SA2- CASSCF(6,6)	800/600	0.39±0.04	53.1±3.0	0.33±0.05	0.81±0.10

Quantum yield and lifetime all from **global switching**

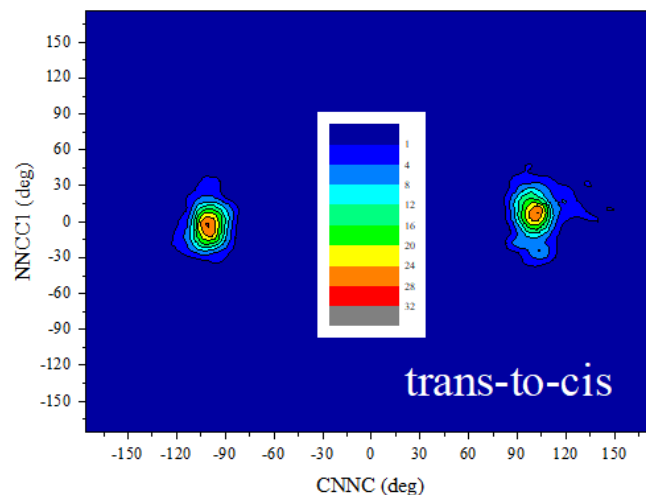
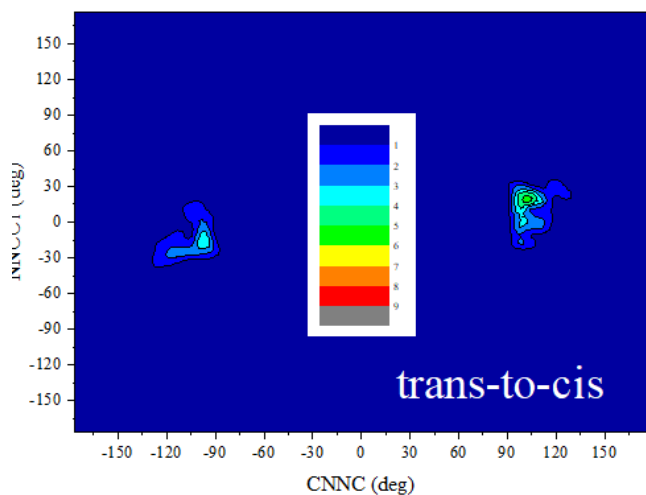
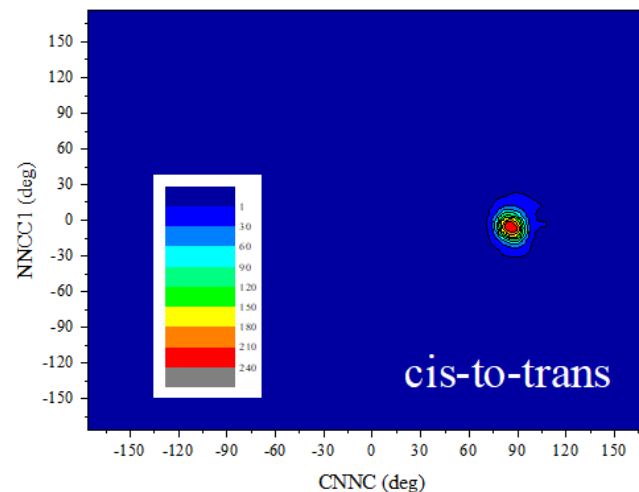
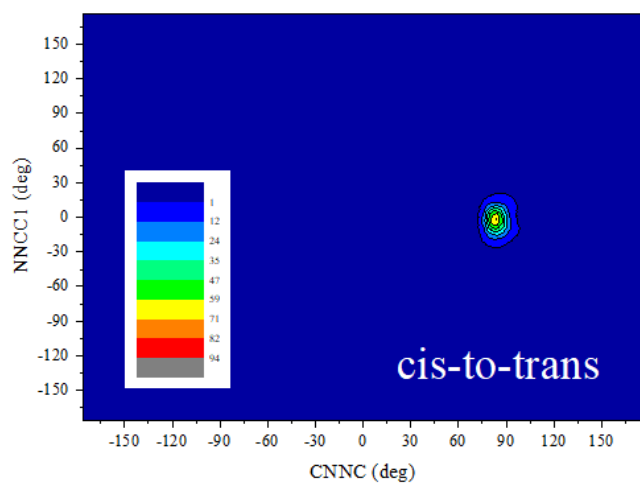




# Hopping spot distributions in terms of CNNC dihedral angle

**SF-TDDFT**

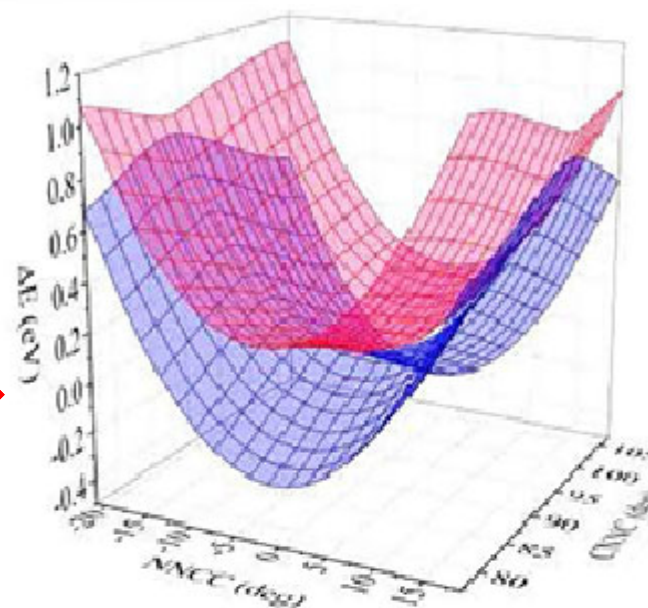
**LR-TDDFT**



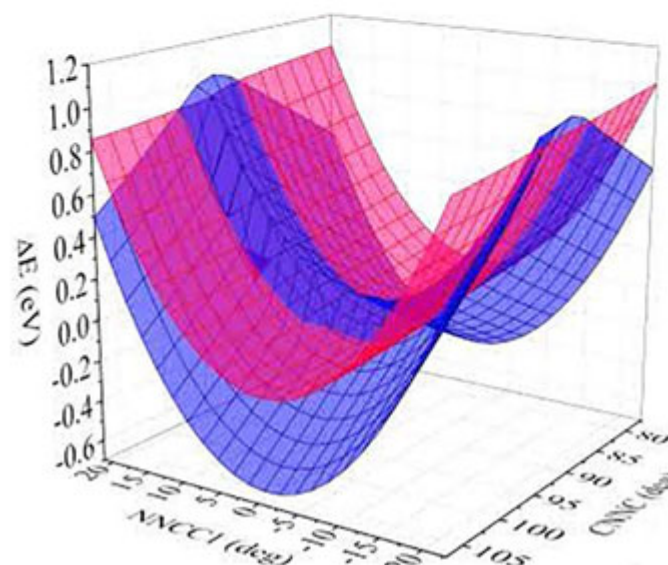


# Conical intersections In SF-TDDFT and TDDFT

**SF-TDDFT: Correctly  
single-cone structure**



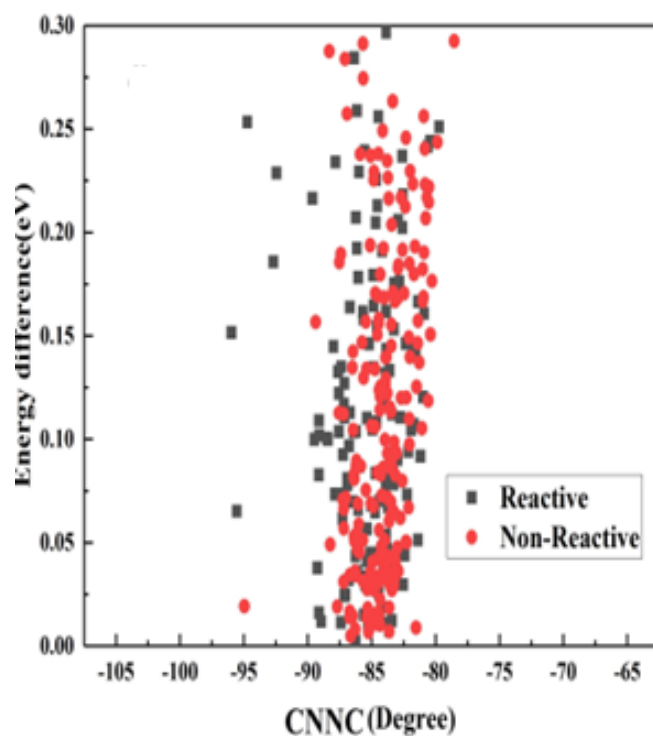
**TDDFT: Artificial  
double-cone structure**



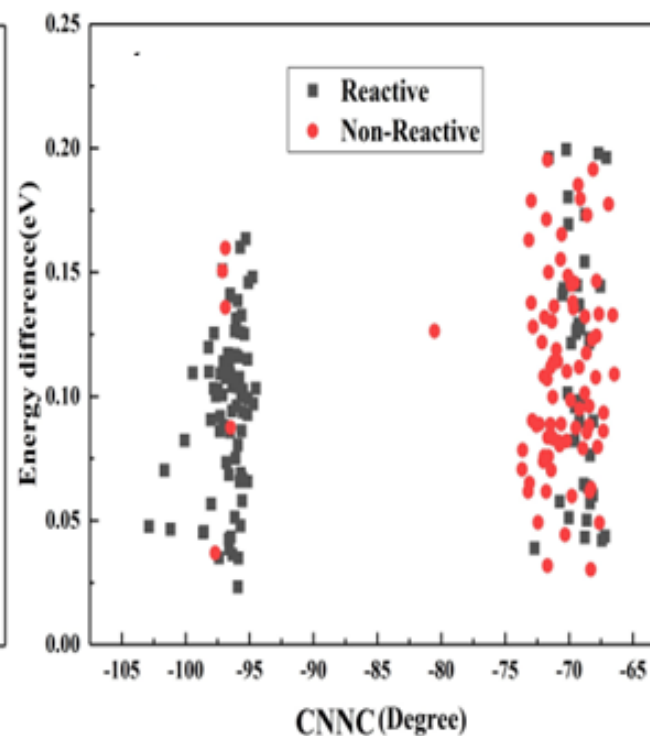
The negative excitation  
energy around CI



# Hopping spot distribution in terms of energy gap



**SF-TDDFT**



**TDDFT**



## Functional and basis set dependence of TDDFT

### TDDFT (Gaussian 16) Dynamics (Our own code)

QY(lifetime)	B3lyp	BHandHLYP	CAM-B3LYP
3-21g	<b>0.51 (35.3)</b>	<b>0.21(36.4)</b>	<b>0.10(36.8)</b>
6-31g	0.57 (37.3)	0.40(36.8)	0.40(37.3)
6-31g(d)	0.63 (47)	0.59(34.6)	0.49(34.4)
cc-pvdz	0.60(37.2)	0.52(34.4)	0.42(33.9)

Average	quantum yield	lifetime
B3LYP	0.6 ( $\pm 5\%$ )	40.5fs ( $\pm 10\%$ )
BHandHLY	0.5 ( $\pm 10\%$ )	35.5fs ( $\pm 4\%$ )
CAM-B3LYP	0.44 ( $\pm 9\%$ )	35.2fs ( $\pm 10\%$ )

### Cis-to-trans

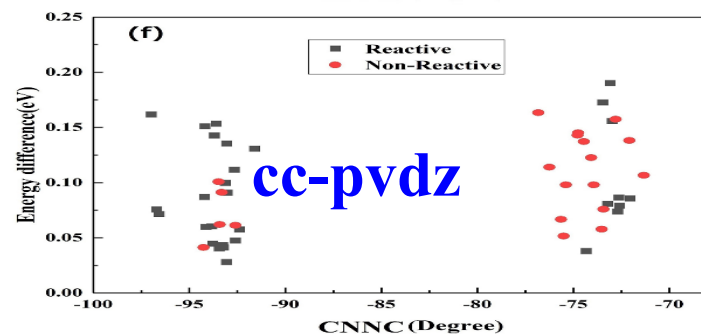
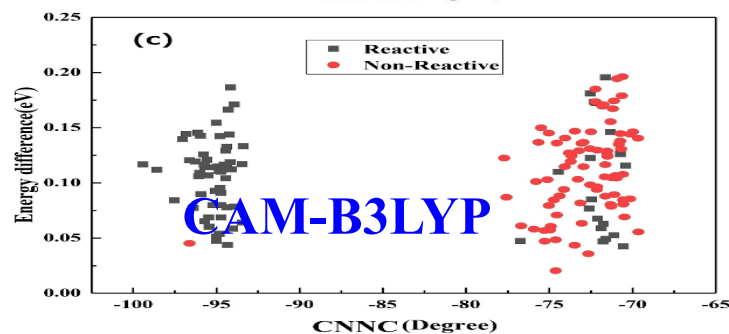
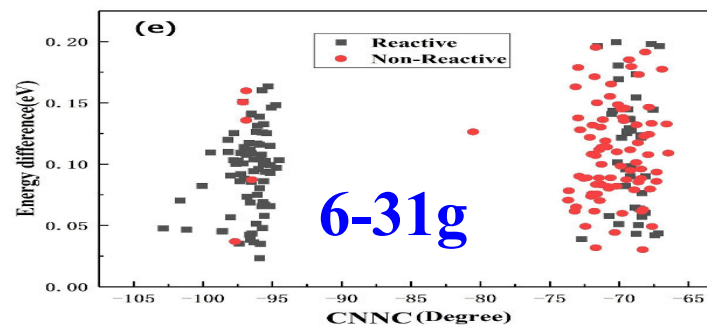
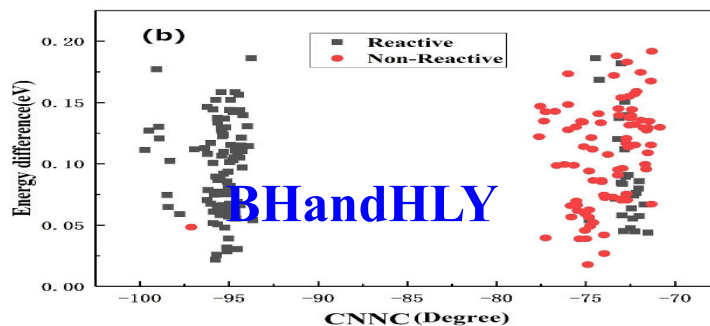
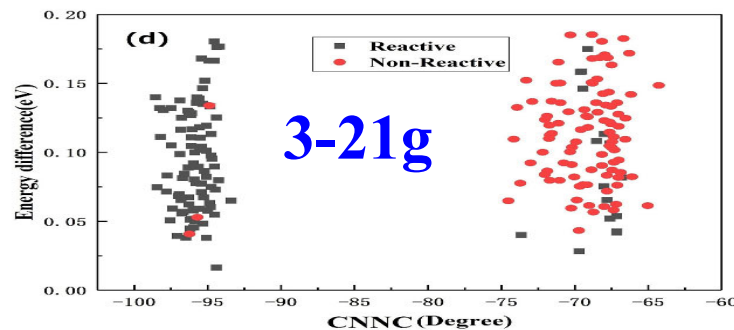
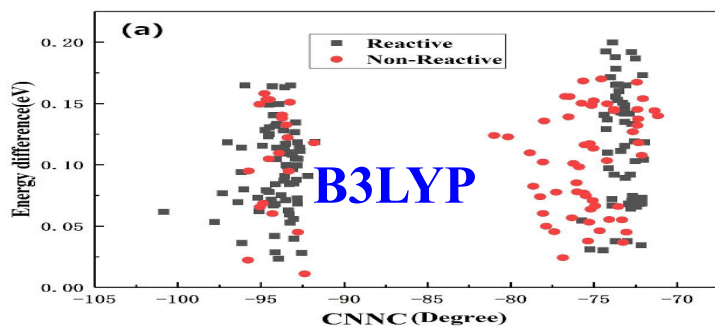
L. Ye, C. Xu, F. L. Gu and C. Zhu, J. Comput. Chem. 41,635 (2020)



## The hopping spot distributions for energy gap

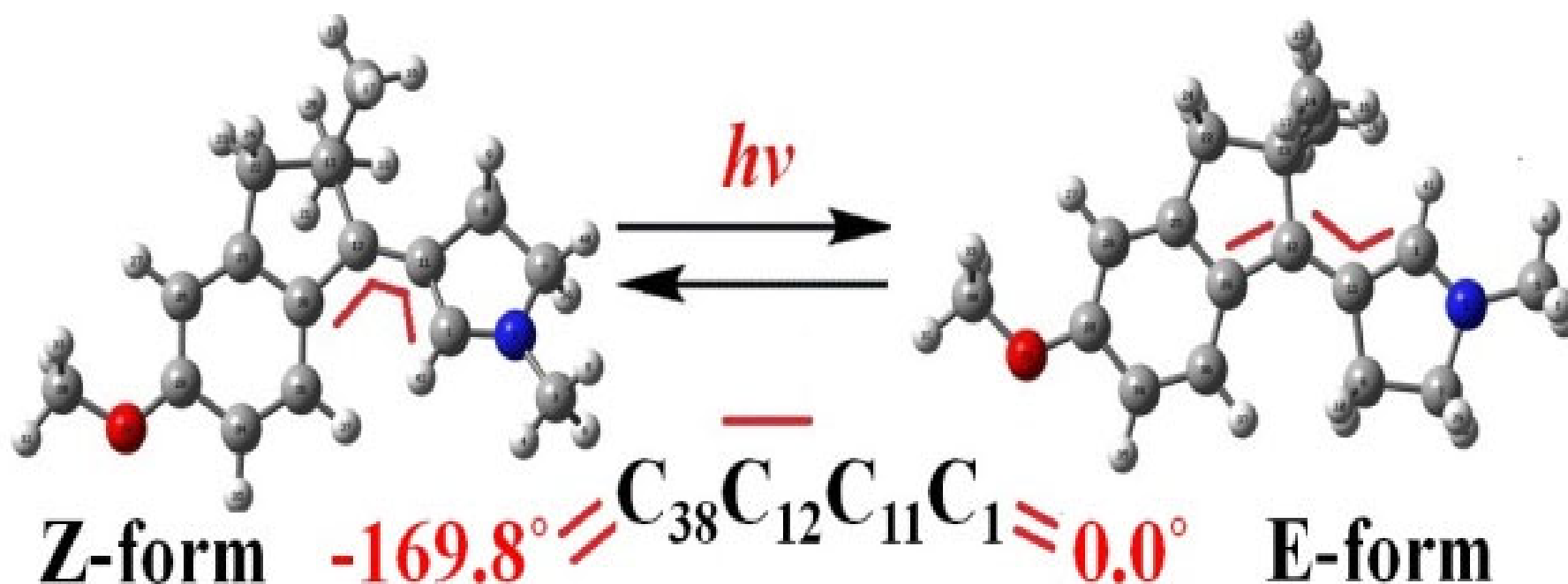
The same 6-31g(d)

The same B3LYP





# 1. dMe-OMe-NAIP photoisomerization





## Photoisomerization of dMe-OMe-NAIP (41 atoms )

- Too big for applying CSSCF method
- Not work with semiempirical method
- The first time with TDDFT

## Unique choice for TDDFT method

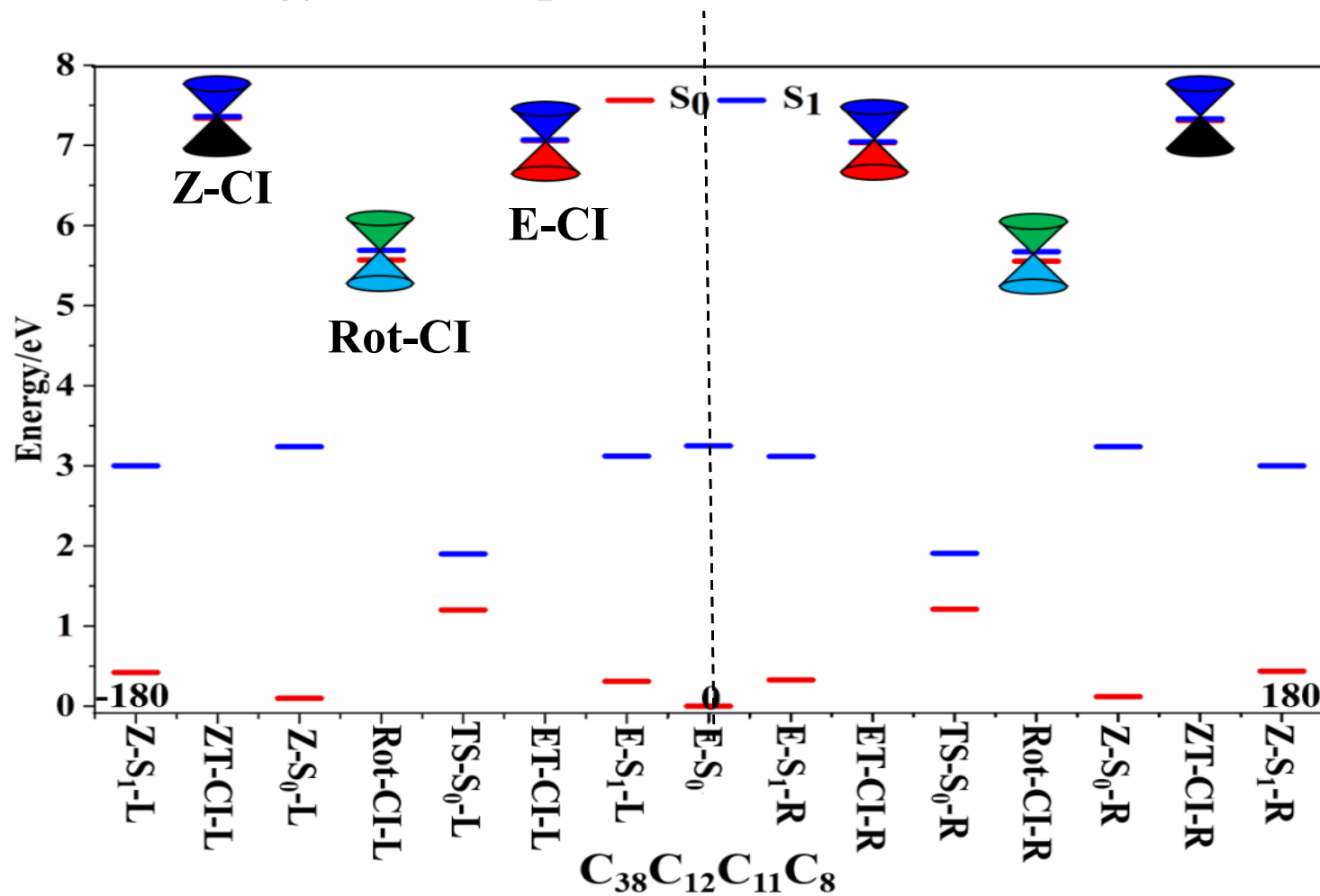
All hopping points are well behaviors,

No problem at all for CI between S0 and S1

**TD-B3LYP/6-31G\***



## Potential energy surface profiles







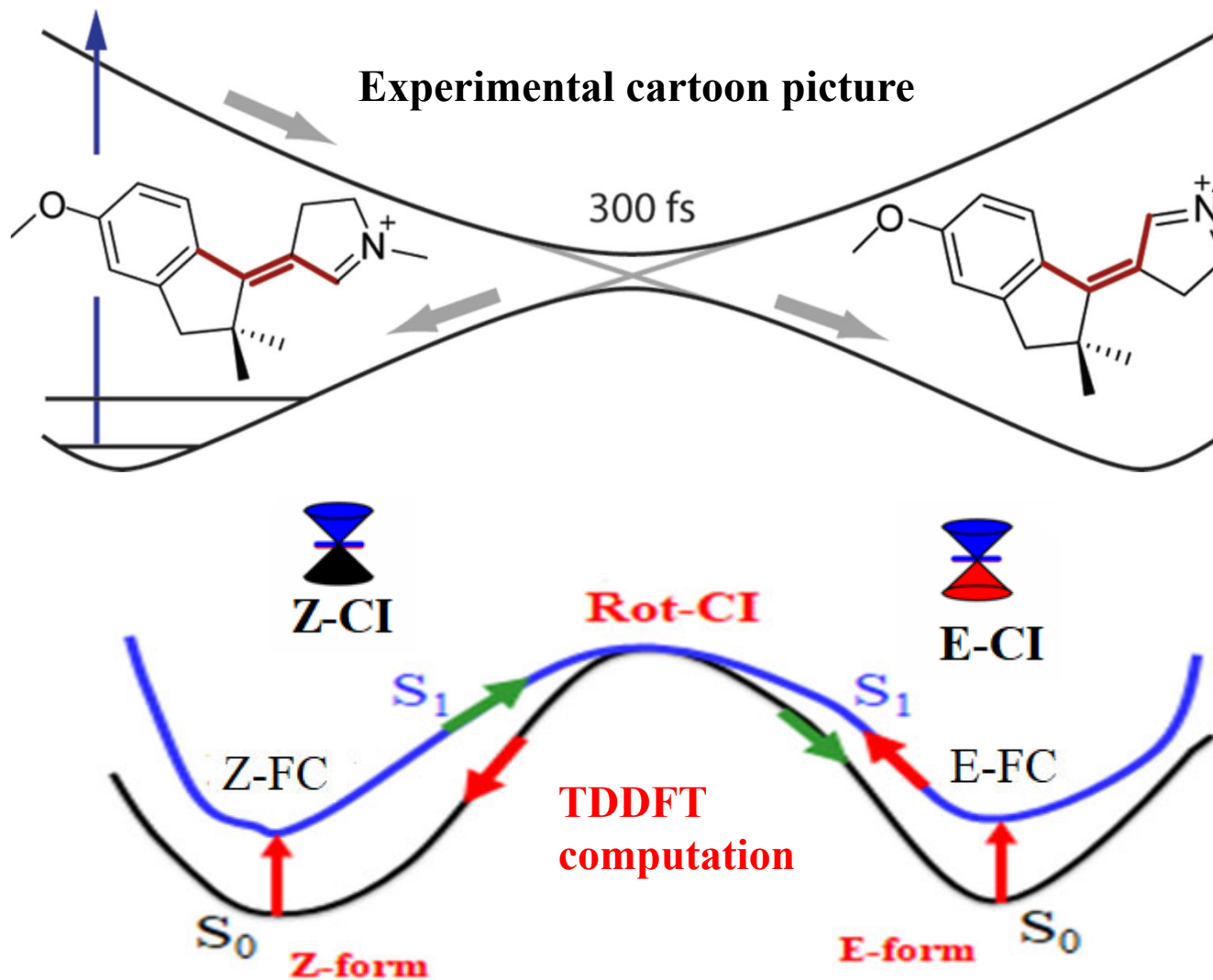
## Simulation results

From E-isomer		
Quantum yield		Lifetime (fs)
0.23	<b>Theory</b>	620
0.25	<b>Exp</b>	~480
0.26	<b>Exp</b>	~480

From Z-isomer		
Quantum yield		Lifetime (fs)
0.15	<b>Theory</b>	600
0.24	<b>Exp</b>	430

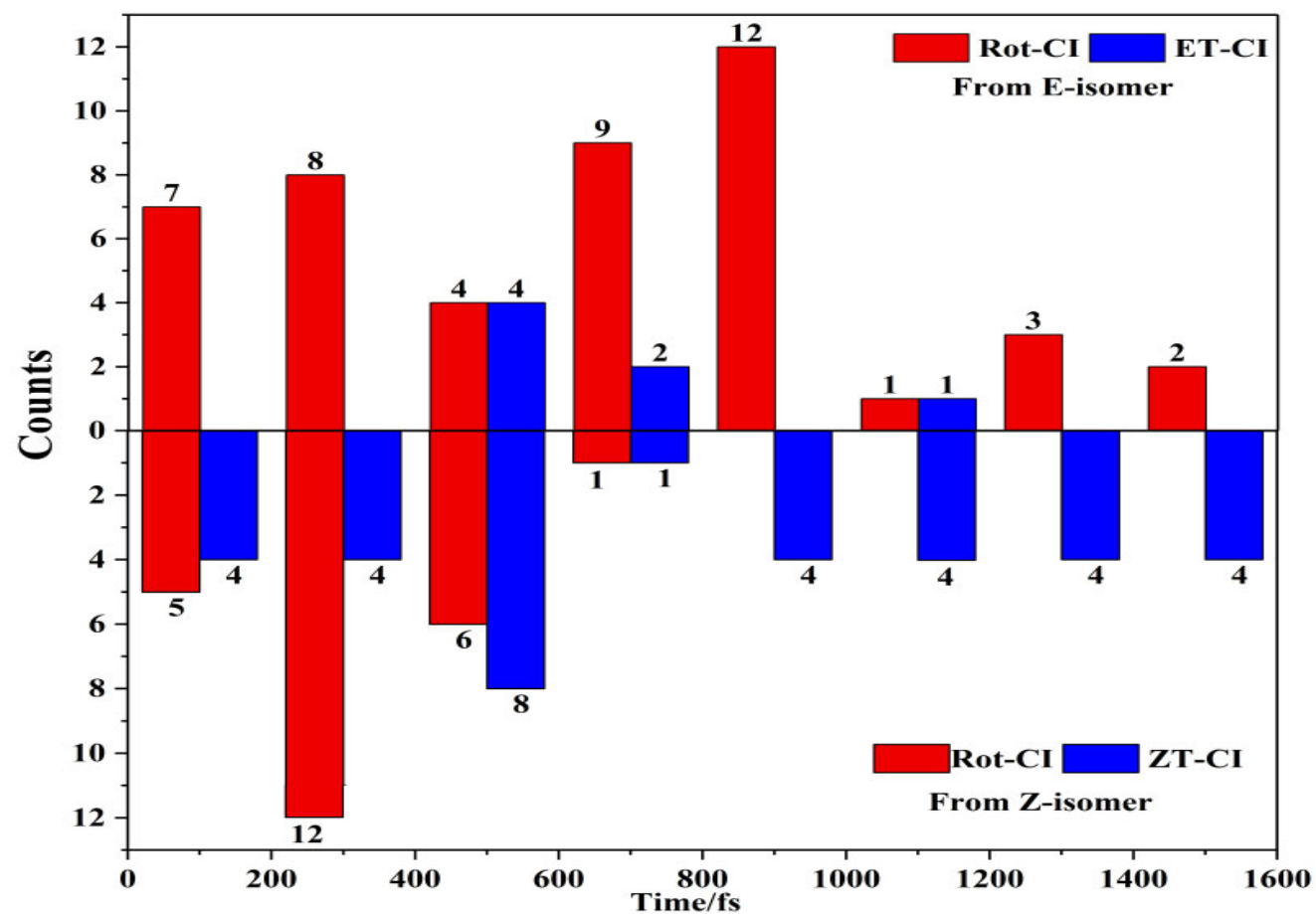


## Photoisomerization mechanism from conical intersection



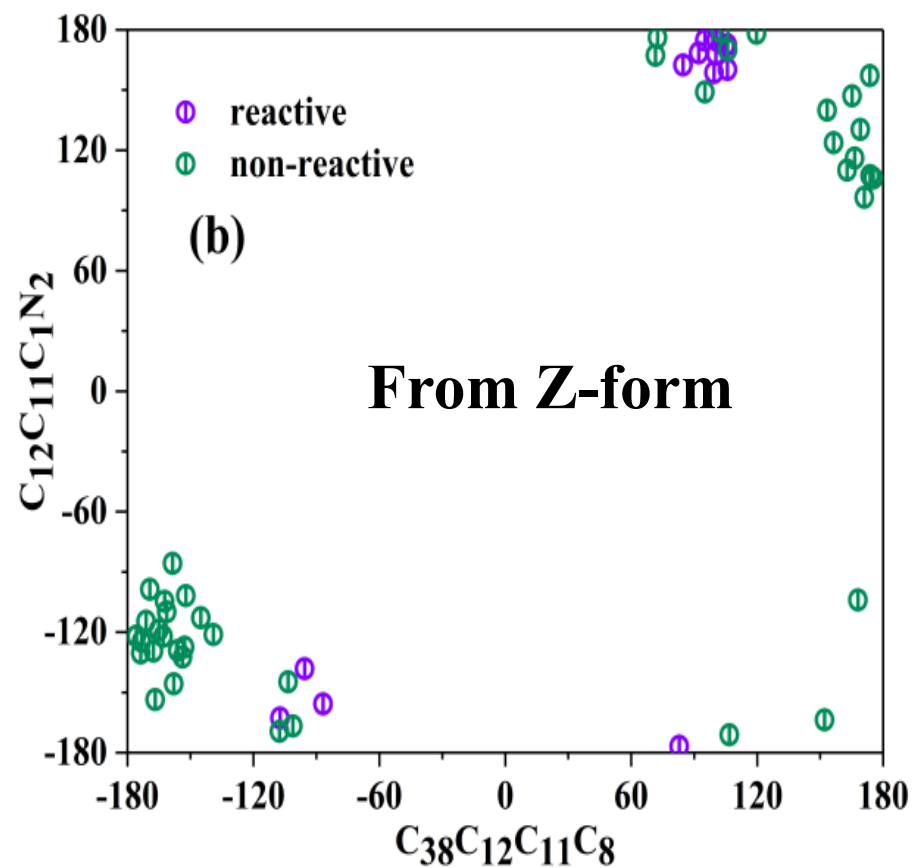
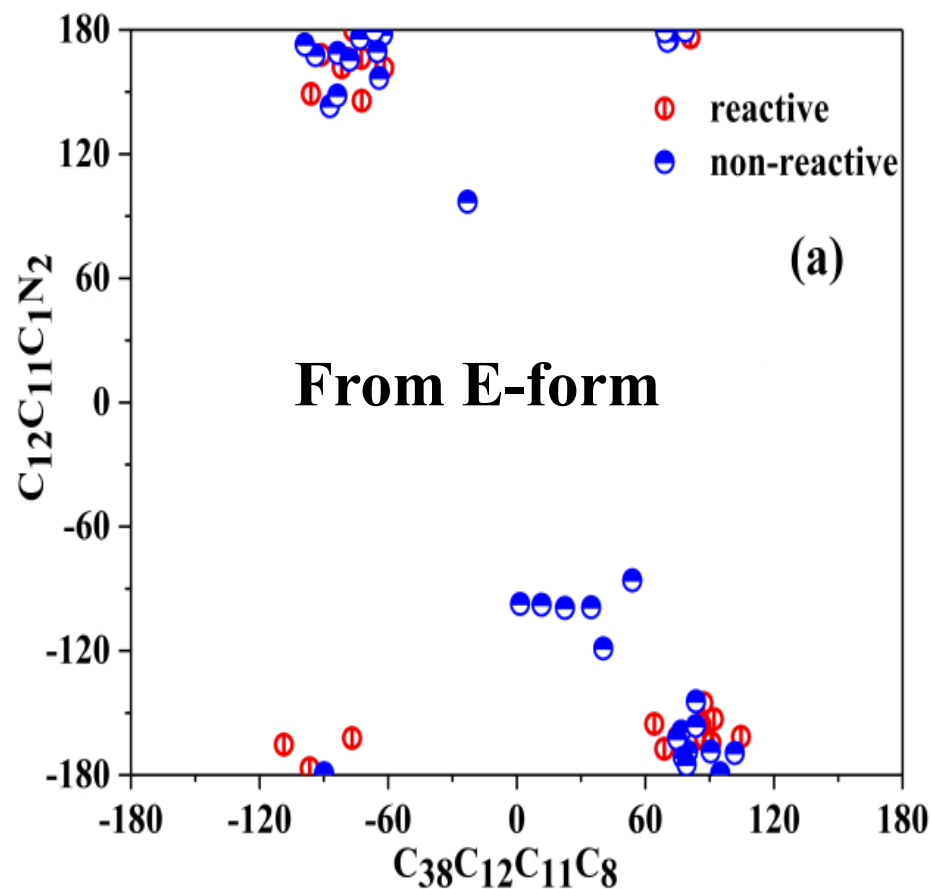


## The number of hopping trajectories via CIs as function of time





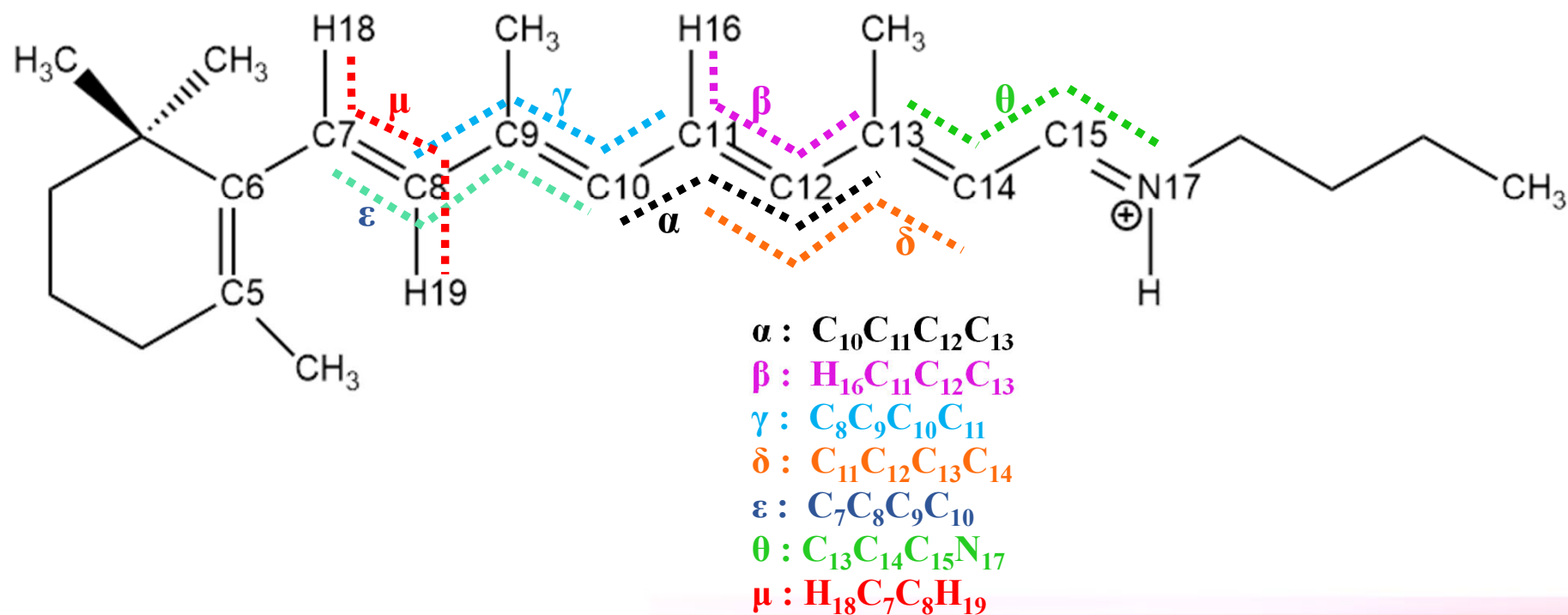
## Hopping spot distributions in terms of CCCC and CCCN



No problem at all hopping points between S0 and S1



## 2. Retinal chromophore isomer photoisomerization





**Photoisomerization of retinal chromophore isomer (63atoms**

**Unique choice for TDDFT method**

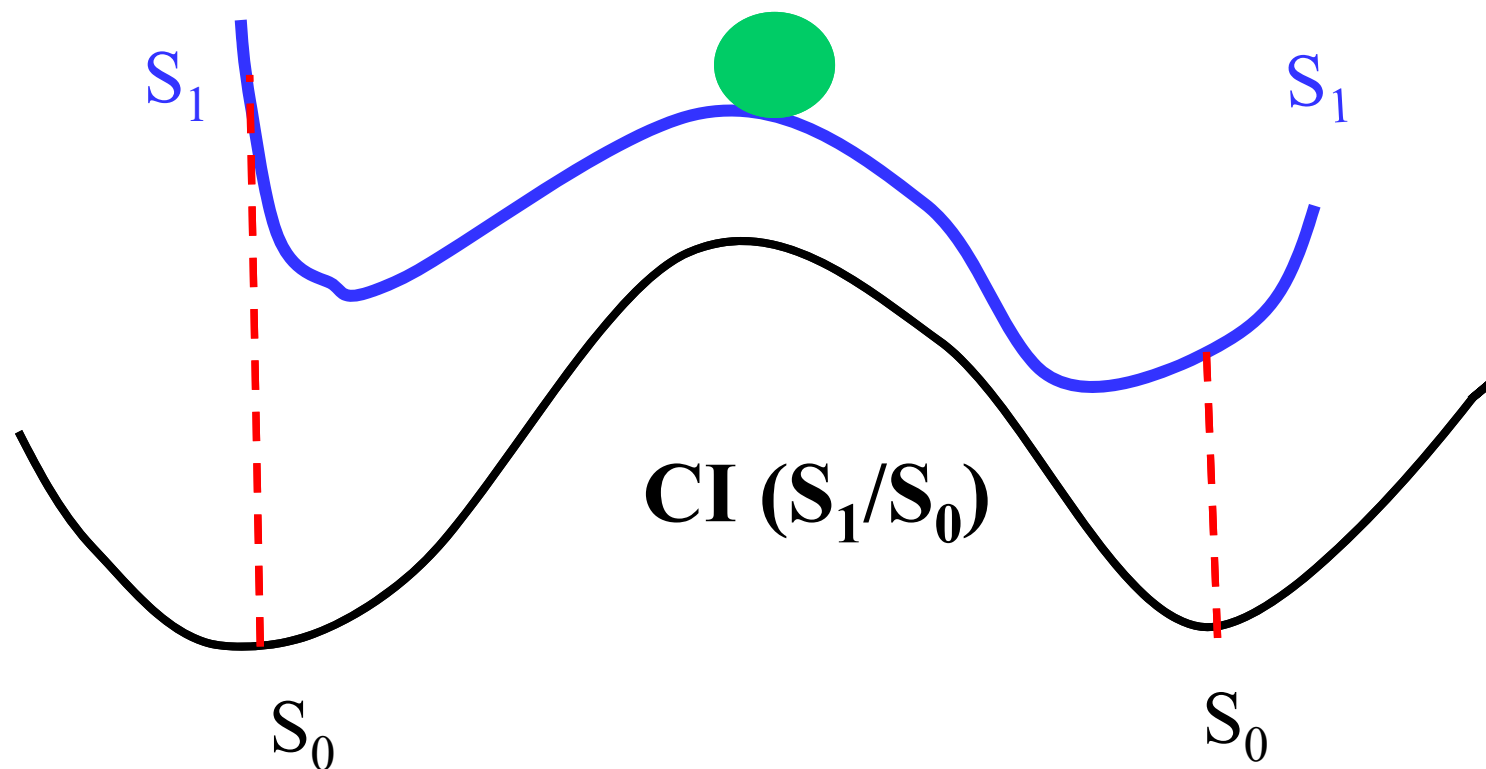
**All hopping points are well behaviors,**

**No problem at all for CI between S0 and S1 again**

**TD-CAM-B3LYP/6-31G**



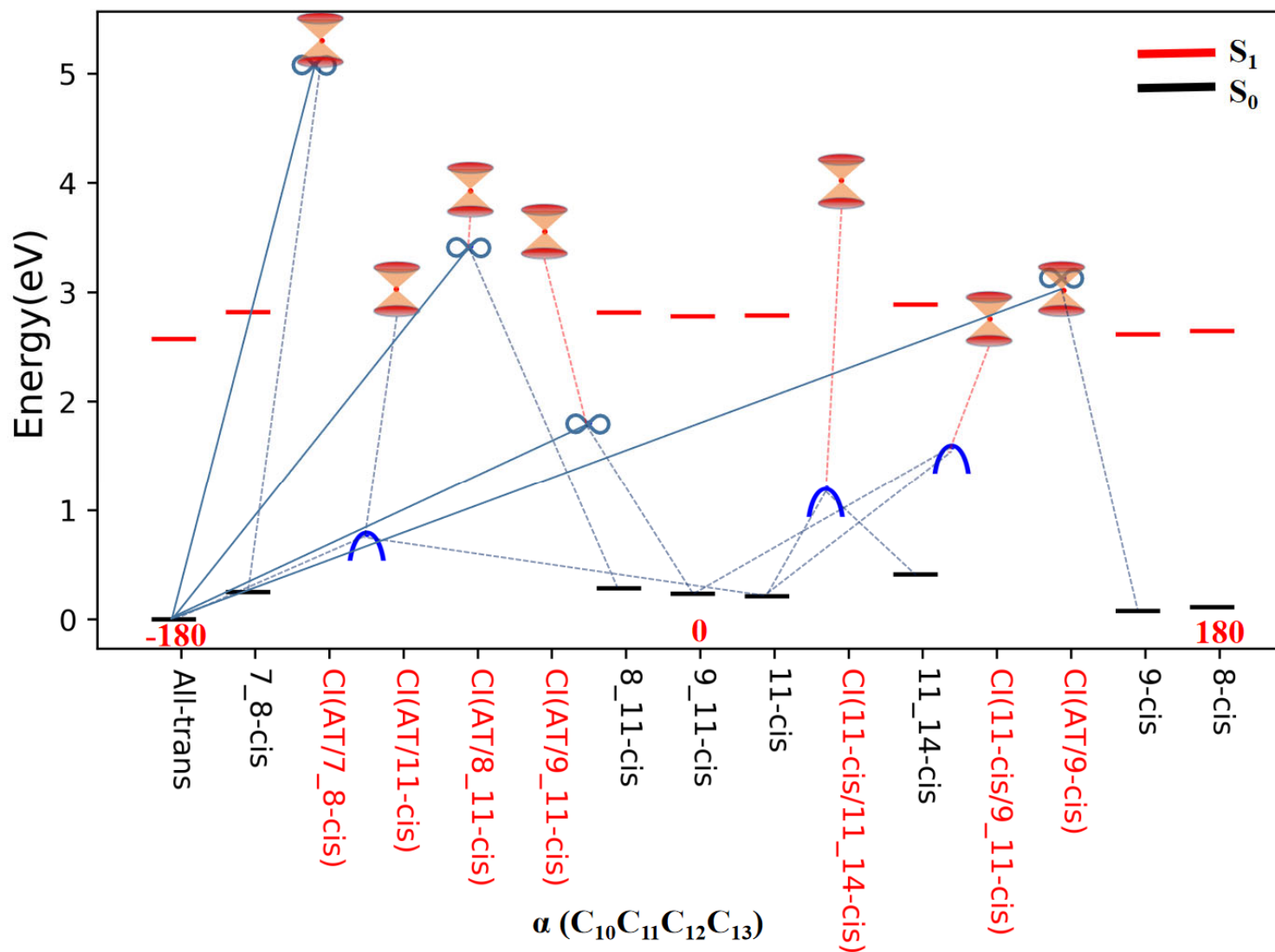
## Trajectory starts at anywhere on potential energy surface



**Global switching with TDDFT in Gaussian**



## Potential energy surface profiles (7 conical intersections)

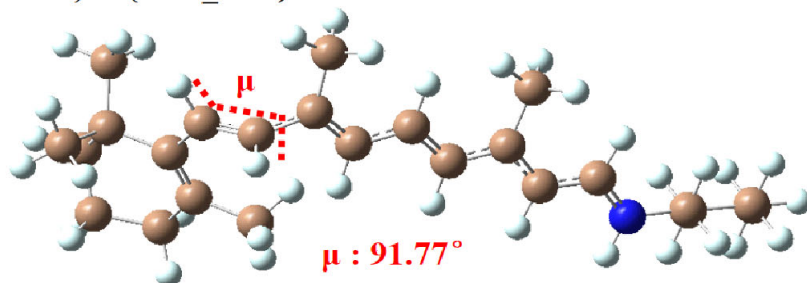




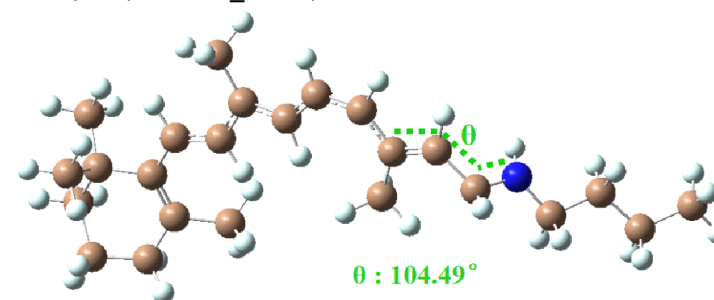


## Geometry structures of conical intersections

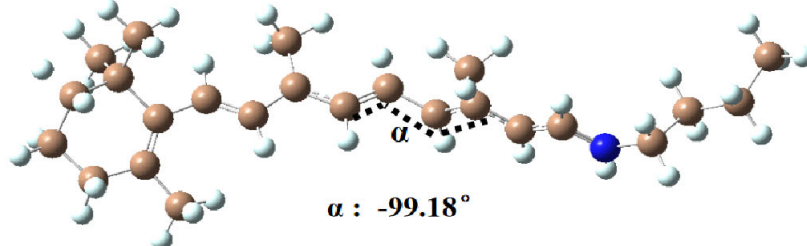
a) CI(AT/7\_8-cis)  $\alpha$ :  $-179.86^\circ$



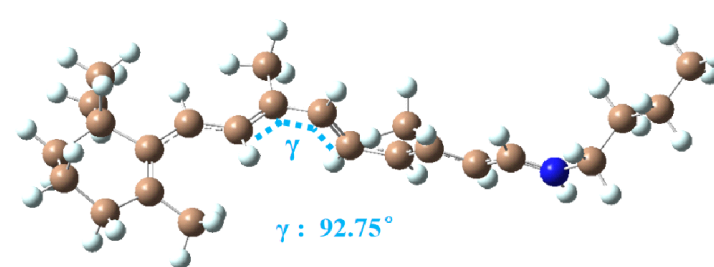
e) CI(11-cis/11\_14-cis)  $\alpha$ :  $1.97^\circ$



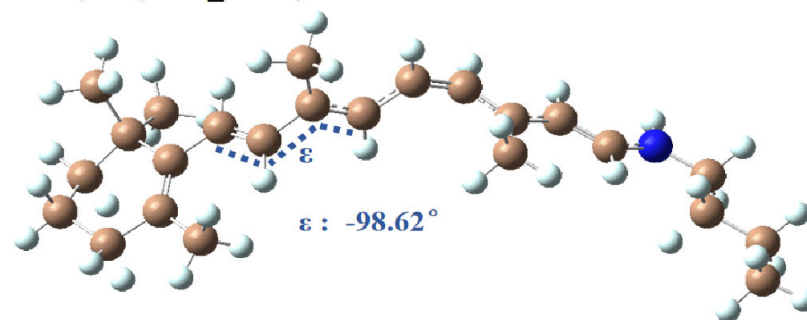
b) CI(AT/11-cis)



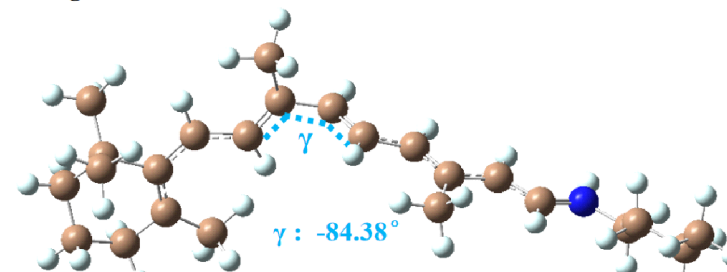
f) CI(11-cis/9\_11-cis)  $\alpha$ :  $23.55^\circ$



c) CI(AT/8\_11-cis)  $\alpha$ :  $-88.74^\circ$

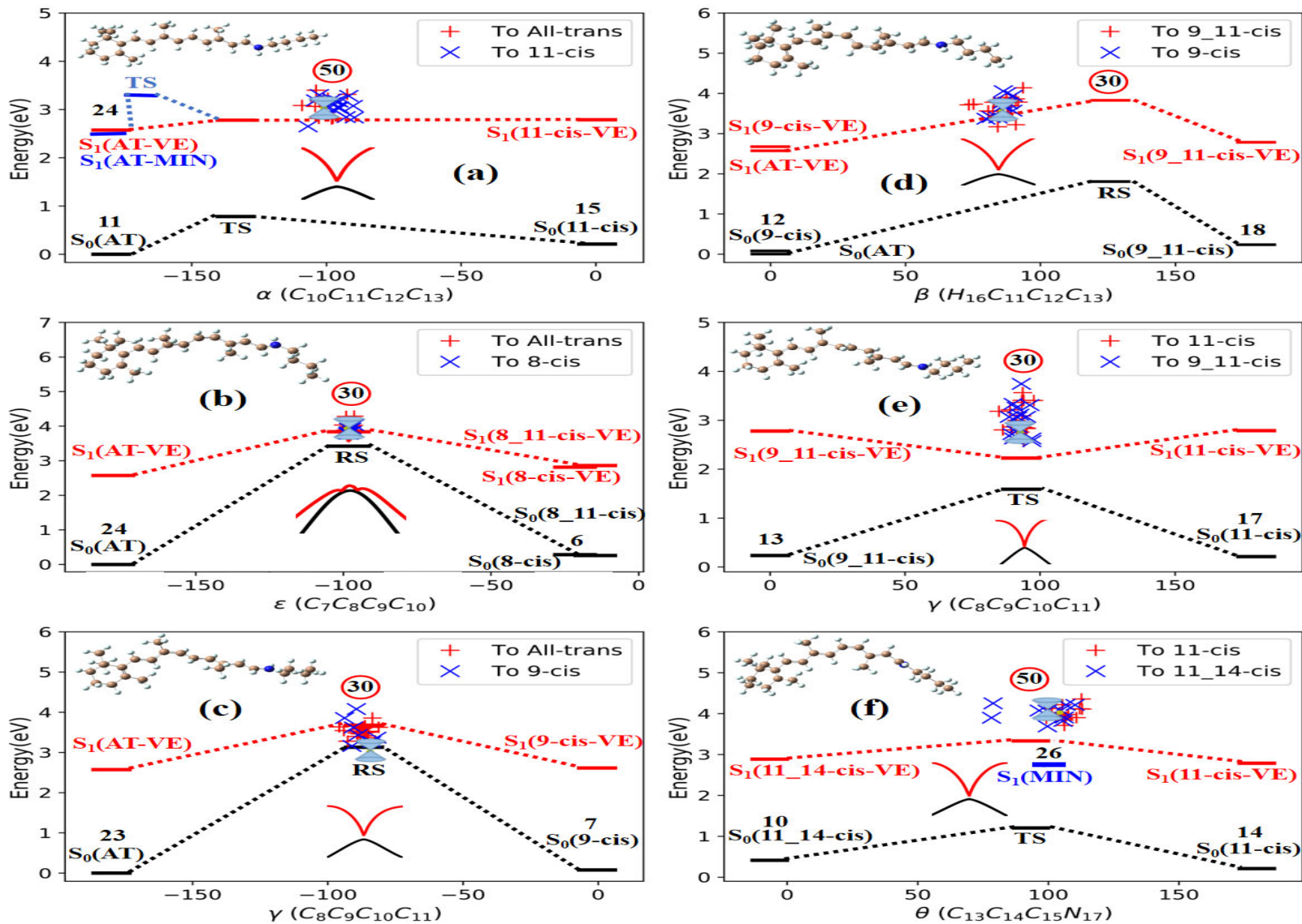


g) CI(AT/9-cis)  $\alpha$ :  $178.61^\circ$



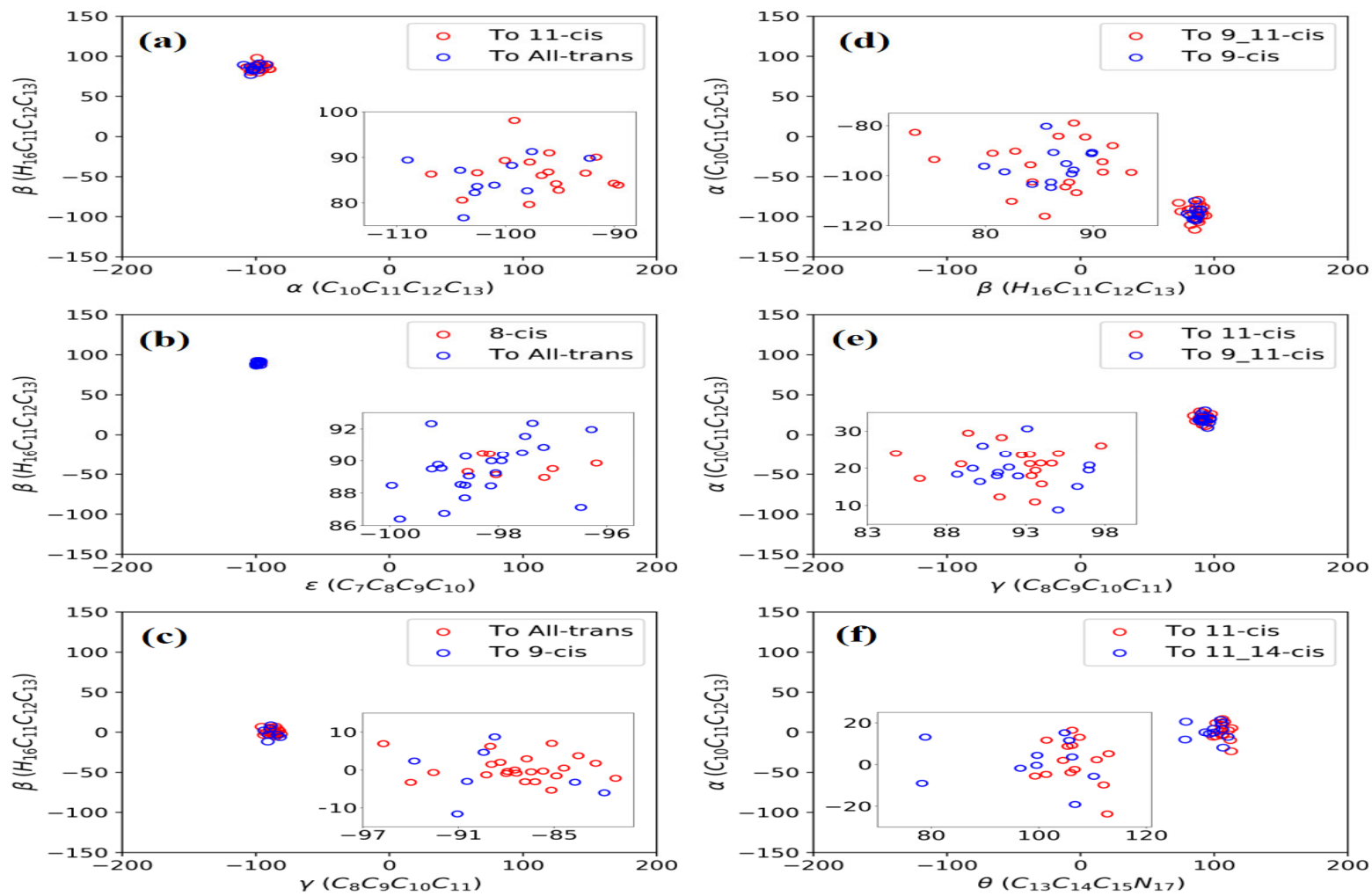


## Hopping points connect to reactants and products





## Hopping spot distributions in terms of dihedral angles





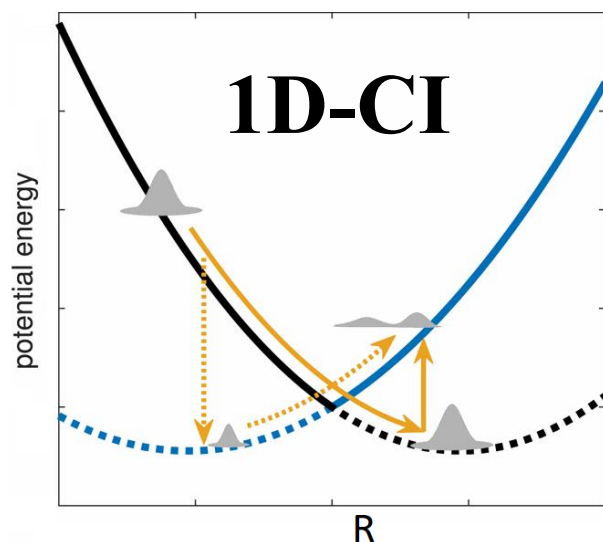
## Simulation results

Product	From all-trans (140)		From 11-cis (130)	
	$N_{\text{traj}}$	QY	$N_{\text{traj}}$	QY
All-trans	58	0.41	11	0.08(0.2 <sup>b</sup> , 0.65 <sup>c</sup> )
<b>11-cis</b>	15	0.11(0.09 <sup>a</sup> )	46	0.35
8-cis	6	0.04		
9-cis	19	0.14(0.02) <sup>a</sup>		
9_11-cis	18	0.13	13	0.1
11_14-cis			10	0.08

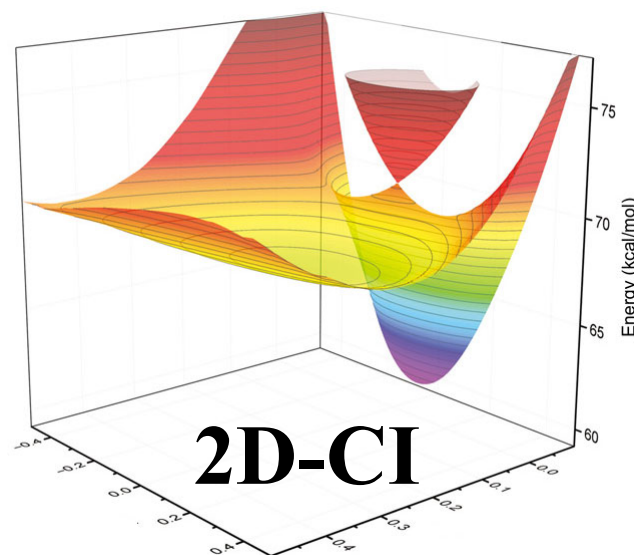
<sup>a</sup>Ref. 2 and <sup>b</sup>Ref. 11 in methanol solution, and <sup>c</sup>Ref. 8 in Rhodopsin protein.



## TD-DFT searching intersection between S0 and S1



Cannot avoid CI (singular point)



Trajectory gets less chance to CI

1. As dimension increases for large system, so that TD-DFT gets better and better
2. As far as trajectory runs not right at CI, TD-DFT OK
3. Energy gap where TD-DFT breaks (system dependent)



# Conclusions

**Global switching TSH method for nonadiabatic simulation**

**No need for calculating nonadiabatic coupling vector**

**No need for calculating seam surfaces**

**Only need calculating potential energy surfaces and its gradients**

**Global switching TSH can also use to search conical intersections**

**Very good to treat CI ( $S_1/S_0$ ) with TDDFT method**

**For large and complicated systems**



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