



THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

VISTA seminar, June 12, 2024

Coupled Quantum Dynamics of Electrons and Protons in Heterogeneous Environments

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First-Principles Method Development for Condensed Matter

Real-time time-dependent density functional theory

for non-equilibrium electron dynamics

Many-body Green's function theory

for electronic excitation properties

Analytical Formulations

$$\frac{\partial}{\partial t} \langle \psi | \rho(t) | \psi \rangle = \langle \psi | \hat{H}(\rho(t)) | \psi \rangle - \langle \psi | \hat{H}(\rho(t)) | \psi \rangle$$
$$\frac{\partial}{\partial t} \langle \psi | \rho(t) | \psi \rangle = \langle \psi | \hat{H}(\rho(t)) | \psi \rangle - \langle \psi | \hat{H}(\rho(t)) | \psi \rangle$$

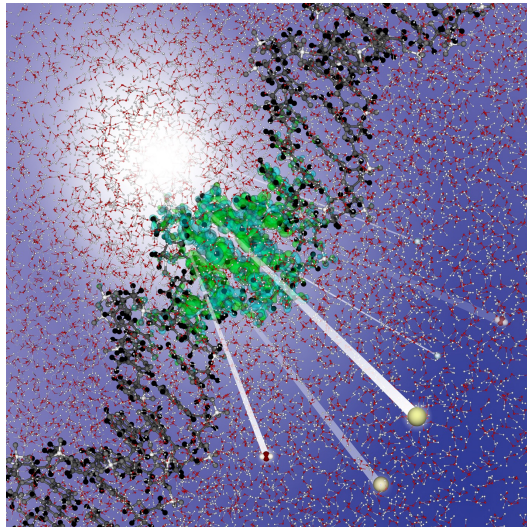
Code Development

```
call invfft('Smooth',vs, dffts)
kedtaus(1:nmrs,iss)=DBLE(vs(1:nmrs))
else
  isup=1
  isdw=2
  do ig=1,nmrs
    vs(nps(ig))=kedtaus(ig,isup)+i*kedtaus(ig,isdw)
    vs(nms(ig))=CONJG(kedtaus(ig,isup))
  end do
  call invfft('Smooth',vs, dffts)
  nm += dr * C phi[1][1]*phi[1][1]*dr*
  kedtaus(1:nmrs,iss)=DBLE(vs(1:nmrs))
  kedtaus(1:nmrs,issdw)=DBLE(vs(1:nmrs,issdw))
endif
if(tpre) then
  rh =
  IF
  do i=1,3
    rhoup(ipol)=grhoCk(ipol,1)
    rhoup(ipol)=grhoCk(ipol,1)
  end do
  CALL tpscc_splin(rh,zeta,grhoup,grhous, &
    atau,sc,v1cup,v1cdw,v2cdw,v3c)
  SC = 0.00
end do
```



Highly-parallelized multi-processor Computers

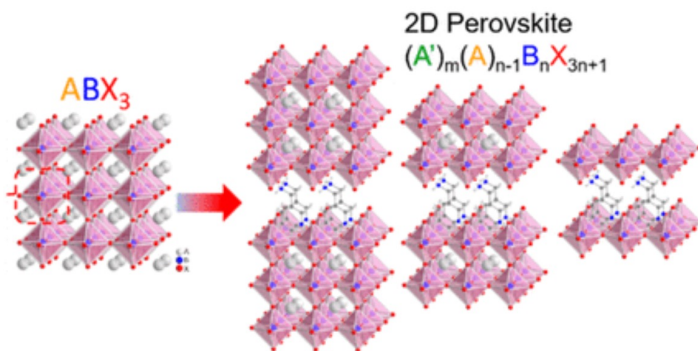
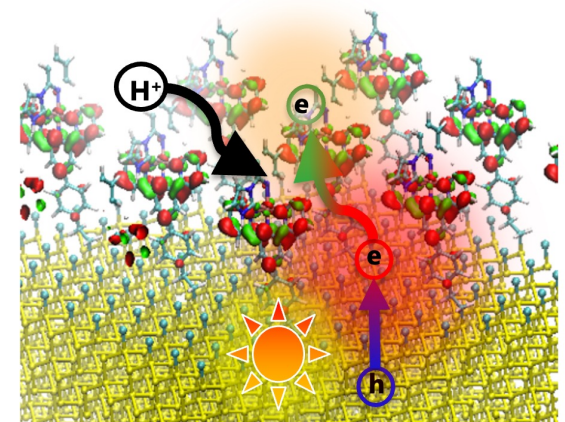
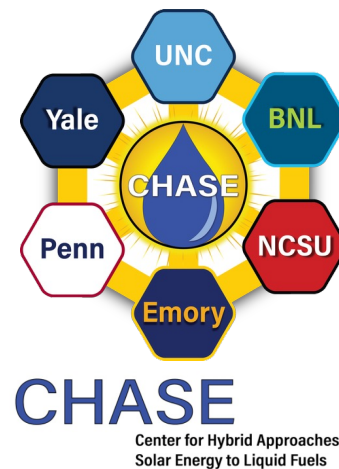
Phenomena of Current Interest



Electronic Stopping

Excitation dynamics under proton irradiation and beyond

Quantum Dynamics of Electrons and Protons
Elucidating the mechanism for CO₂ conversion



Novel Materials and Dynamics

Application of first-principles method to “explore” interesting materials and dynamic phenomena

Kanai Group @ UNC



THE UNIVERSITY
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CHASE

Center for Hybrid Approaches in
Solar Energy to Liquid Fuels



Dr. Jianhang Xu

Dr. Chris Shepard

Dr. Ruiyi Zhou

Sampreeti Bhattacharya

John Bost

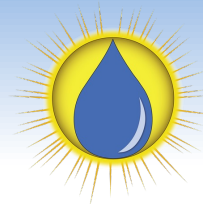
Tom Carney

Nicolas Boyer

Kiichi Ogata



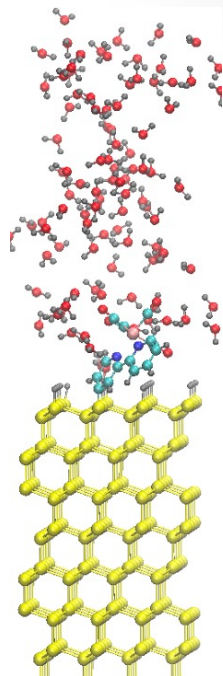
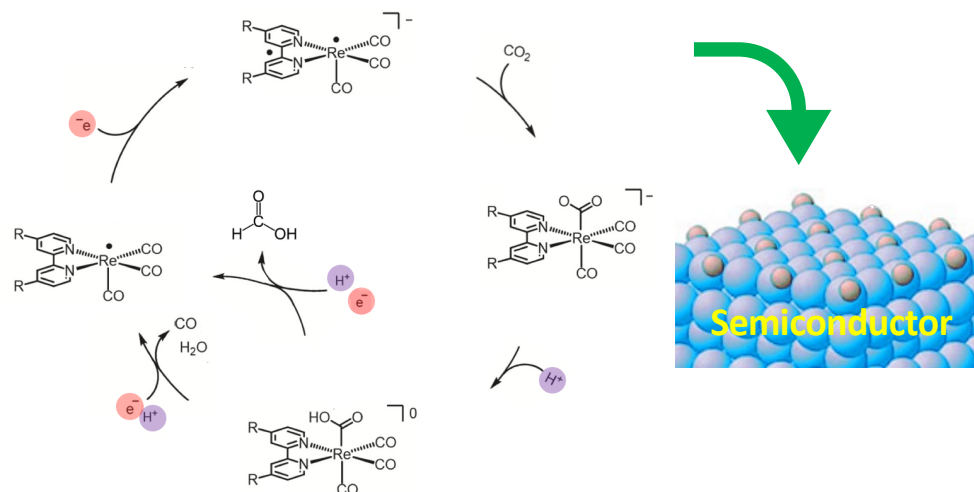
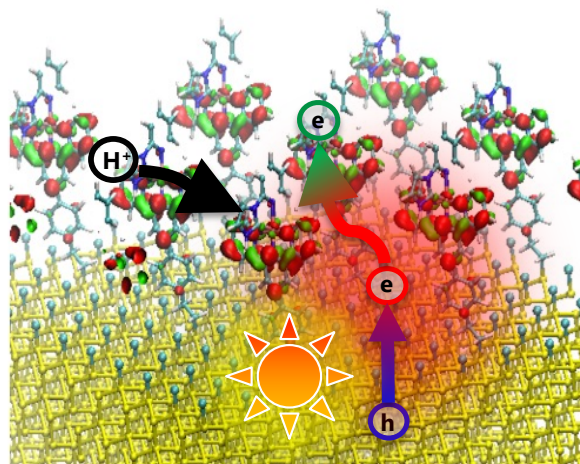
Theory Challenges in DOE Hub: CHASE



CHASE

Center for Hybrid Approaches in
Solar Energy to Liquid Fuels

Development of CO₂-catalyst functionalized semiconductor photoelectrodes



How does surface adsorption impact H⁺/e⁻ transfer steps?

Complex heterogeneous environments need to be modeled.

To what extent is the interfacial excited e⁻ transfer coupled with H⁺ addition at an interface?

Coupled quantum dynamics of protons and electrons need to be considered.

Multicomponent DFT

$$E[\rho^e, \rho^p] = E_{\text{ref}}[\rho^e, \rho^p] + E_{XC}^e[\rho^e] + E_{XC}^p[\rho^p] + E_{\text{epc}}[\rho^e, \rho^p]$$

“Non-Born–Oppenheimer Density Functional Theory of Molecular Systems”

J. F. Capitani, R. F. Nalewajski, and R. G. Parr, J. Chem. Phys. 76, 568 (1982)

“Multicomponent Density-Functional Theory for Electrons and Nuclei”

T. Kreibich and E. K. U. Gross, Phys. Rev. Lett. 86, 2984 (2001)

Nuclear Electronic Orbital (NEO) method

F. Pavosevic, T. Culpitt, S. Hammes-Schiffer, Chem. Rev. **120**, 4222 (2020)

NEO to KS-DFT for periodic systems

J. Xu, R. Zhou, Z. Tao, C. Malbon, V. Blum, S. Hammes-Schiffer, Y. Kanai, J. Chem. Phys. 156, 224111 (2022)

$$\psi_{i,\mathbf{k}}^e(\mathbf{r}^e) = \sum_{\mu} c_{i\mu,\mathbf{k}} \sum_N e^{i\mathbf{k}\cdot\mathbf{T}(\mathbf{N})} \phi_{\mu}^{e:NAO/GTO}(\mathbf{r}^e - \mathbf{R}_{\mu} + \mathbf{T}(\mathbf{N}))$$

$$\psi_i^p(\mathbf{r}^p) = \sum_m c_{im} \sum_N \phi_m^{p:GTO}(\mathbf{r}^p - \mathbf{R}_m + \mathbf{T}(\mathbf{N}))$$

Atom-centered basis functions

KS Hamiltonian in Multicomponent DFT

$$\hat{H}_{KS}^e = -\frac{1}{2}\nabla_e^2 + v_{ext}(\mathbf{r}^e) + v_{es}^e(\mathbf{r}^e) - v_{es}^p(\mathbf{r}^e) + \frac{\delta E_{XC}^e[\rho^e]}{\delta\rho^e} + \frac{\delta E_{epc}[\rho^e, \rho^p]}{\delta\rho^e}$$

$$\hat{H}_{KS}^p = -\frac{1}{2M^p}\nabla_p^2 - v_{ext}(\mathbf{r}^p) - v_{es}^e(\mathbf{r}^p) + v_{es}^p(\mathbf{r}^p) + \frac{\delta E_{XC}^p[\rho^p]}{\delta\rho^p} + \frac{\delta E_{epc}[\rho^e, \rho^p]}{\delta\rho^p}$$

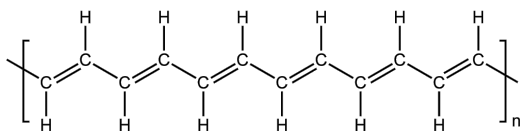
Electron-Proton Correlation

Multicomponent extensions of **Colle-Salvetti formalism** have been developed. The simplest form is the local density approximation (LDA)

Y. Yang, et al., J. Chem. Phys. **147**, 114113 (2017)

$$E_{epc}[\rho^e, \rho^p] \approx E_{epc17-2}[\rho^e, \rho^p] = - \int \frac{\rho^e(\mathbf{r})\rho^p(\mathbf{r})}{a + b\rho^e(\mathbf{r})^{\frac{1}{2}}\rho^p(\mathbf{r})^{\frac{1}{2}} + c\rho^e(\mathbf{r})\rho^p(\mathbf{r})} d\mathbf{r}$$

Proof-of-Principle Demonstration



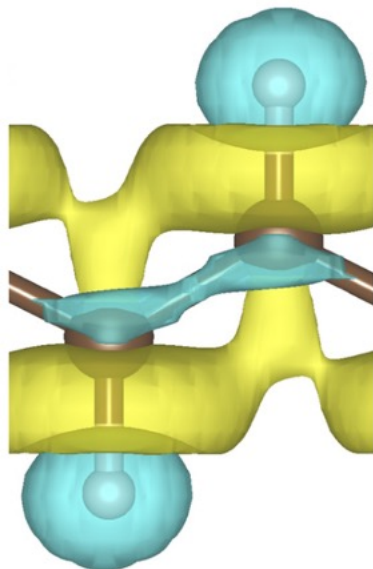
NEO-induced Electron
Density difference

(+) change

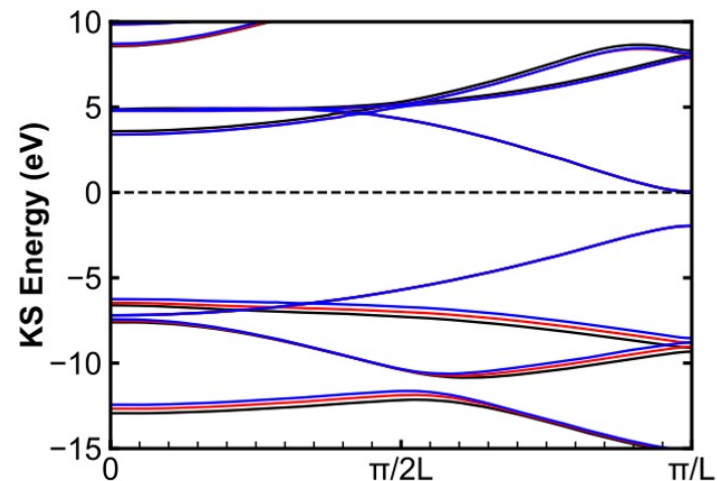
(-) change

Isosurface @ $1.7 \times 10^{-2} \text{ \AA}^{-3}$

$$\Delta\rho^e(\mathbf{r})$$



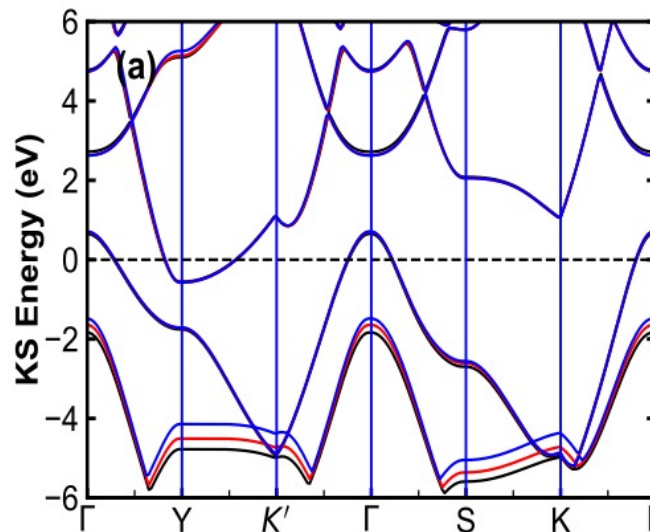
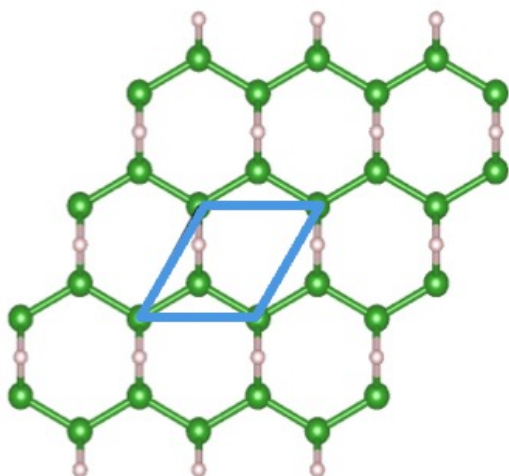
Band Structure



— DFT
— NEO-DFT, no epc
— NEO-DFT, epc17-2

Brillouin Zone

Hydrogen Boride 1D sheet : B-H-B 3-center-2-electron bonds



DFT
NEO-DFT w/o epc
NEO-DFT w/ epc

Dynamics : Lagrangian for our NEO DFT-KS System

$$\begin{aligned}
 L^{NEO}(t) = & \int d\mathbf{r}^e \int d\mathbf{k} \sum_j [\psi_{j\mathbf{k}}^e(\mathbf{r}^e, t)]^* \left[i \frac{\partial}{\partial t} + \frac{1}{2m^e} \nabla_{\mathbf{r}^e}^2 \right] \psi_{j\mathbf{k}}^e(\mathbf{r}^e, t) \\
 & - \frac{1}{2} \iint d\mathbf{r}^e d\mathbf{r}^{e'} \frac{e^2}{|\mathbf{r}^e - \mathbf{r}^{e'}|} \rho^e(\mathbf{r}^e, t) \rho^e(\mathbf{r}^{e'}, t) - E_{XC}^e[\rho^e] \\
 & + \int d\mathbf{r}^p \sum_j [\psi_j^p(\mathbf{r}^p, t)]^* \left[i \frac{\partial}{\partial t} + \frac{1}{2M^p} \nabla_{\mathbf{r}^p}^2 \right] \psi_j^p(\mathbf{r}^p, t) \\
 & - \frac{1}{2} \iint d\mathbf{r}^p d\mathbf{r}^{p'} \frac{e^2}{|\mathbf{r}^p - \mathbf{r}^{p'}|} \rho^p(\mathbf{r}^p, t) \rho^p(\mathbf{r}^{p'}, t) - E_{XC}^p[\rho^p] \\
 & + \frac{1}{2} \iint d\mathbf{r}^e d\mathbf{r}^p \frac{e^2}{|\mathbf{r}^e - \mathbf{r}^p|} \rho^e(\mathbf{r}^e, t) \rho^p(\mathbf{r}^p, t) - E_{epc}[\rho^e, \rho^p] \\
 & + \sum_I \frac{1}{2} M_I [\dot{\mathbf{R}}_I(t)]^2 - \sum_{I < J} \frac{Z_I Z_J e^2}{|\mathbf{R}_I(t) - \mathbf{R}_J(t)|} \\
 & - \int d\mathbf{r}^p \rho^p(\mathbf{r}^p, t) \sum_I \frac{Z_I e^2}{|\mathbf{r}^p - \mathbf{R}_I(t)|} + \int d\mathbf{r}^e \rho^e(\mathbf{r}^e, t) \sum_I \frac{Z_I e^2}{|\mathbf{r}^e - \mathbf{R}_I(t)|}
 \end{aligned}$$

TD-KS equations for RT-NEO-TDDFT

$$A = \int L^{NEO}(\mathbf{R}, \dot{\mathbf{R}}, \{\psi_{i,\mathbf{k}}^e\}, \{\psi_I^p\}, t) dt \quad \frac{\delta A}{\langle \delta \psi^x |} = 0 \quad \frac{\delta L^{NEO}}{\langle \delta \psi^x |} - \frac{d}{dt} \frac{\delta L^{NEO}}{\langle \delta \dot{\psi}^x |} = 0$$

See. e.g. Kramer and Saraceno, "Geometry of the Time-Dependent Variational Principle" (Springer, Berlin, 1981).

Electrons:
$$i \frac{\partial}{\partial t} \psi_{i,\mathbf{k}}^e(\mathbf{r}^e, t) = \left[-\frac{1}{2} \nabla_i^2 + v_{\text{DFT-KS}}^e(\mathbf{r}^e) - v_{\text{es}}^p(\mathbf{r}^e) + \frac{\delta E_{\text{epc}}[\rho^e, \rho^p]}{\delta \rho^e} \right] \psi_{i,\mathbf{k}}^e(\mathbf{r}^e, t)$$

RT-TDDFT

All-electron real-time and imaginary-time TDDFT within a numeric atom-centered basis function framework J. Hecke, et al. [J. Chem. Phys. 155, 154801 \(2021\)](https://doi.org/10.1063/1.5044441)

Protons:
$$i \frac{\partial}{\partial t} \psi_I^p(\mathbf{r}^p, t) = \left[-\frac{1}{2M^p} \nabla_I^2 + v_{\text{DFT-KS}}^p(\mathbf{r}^p) - v_{\text{es}}^e(\mathbf{r}^p) + \frac{\delta E_{\text{epc}}[\rho^e, \rho^p]}{\delta \rho^p} \right] \psi_I^p(\mathbf{r}^p, t)$$

L. Zhao, et al., J. Phys. Chem. Lett. **11**, 4052 (2020)

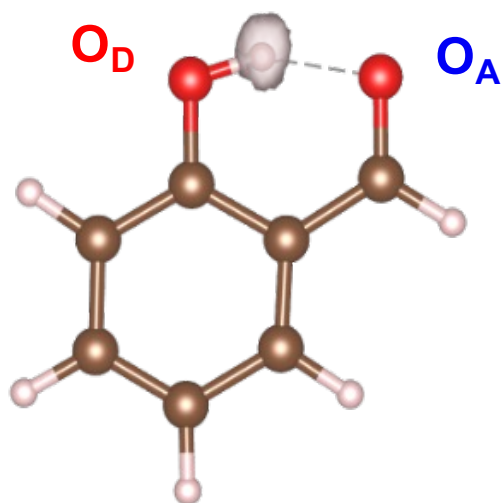
$$\psi_i^p(\mathbf{r}^p, t) = \sum_m c_{im}(t) \sum_N \phi_m^{p:GTO}(\mathbf{r}^p - \mathbf{R}_m + \mathbf{T}(\mathbf{N}))$$

$$\mathbf{C}(t + \Delta t) = \mathbf{S}^{-\frac{1}{2}} \exp\left(-i\Delta t \mathbf{S}^{-\frac{1}{2}} \mathbf{H}_{\text{KS}}\left(t + \frac{\Delta t}{2}\right) \mathbf{S}^{-\frac{1}{2}}\right) \mathbf{S}^{\frac{1}{2}} \mathbf{C}(t)$$

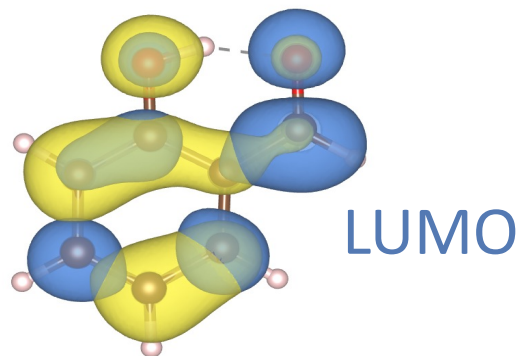
$$\exp(\mathbf{A}) = \mathbf{V} \text{diag}(e^{\lambda_1}, e^{\lambda_2} \dots e^{\lambda_n}) \mathbf{V}^{-1}$$

Electronic Excitation Induced Intra-Molecular Proton Transfer

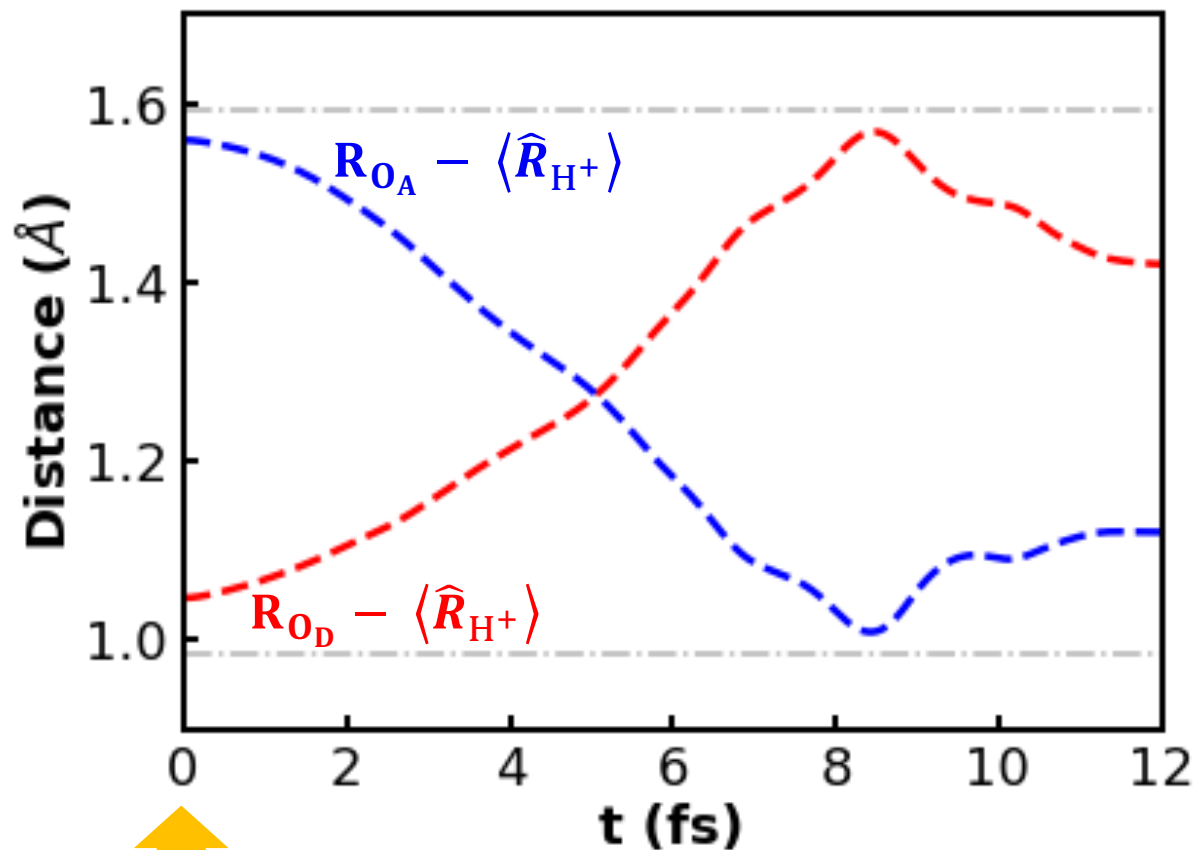
oHBA molecule



HOMO



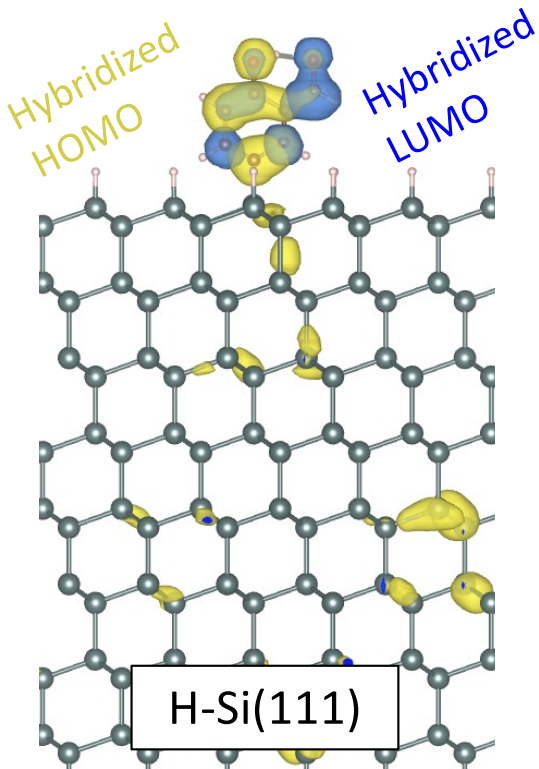
Electrons: PBE + Tier2 NAO basis
Protons: HF + 4s4p basis w/ 3 "ghost" centers
EPC: epc-17-2 , dt=4.8 attosecs



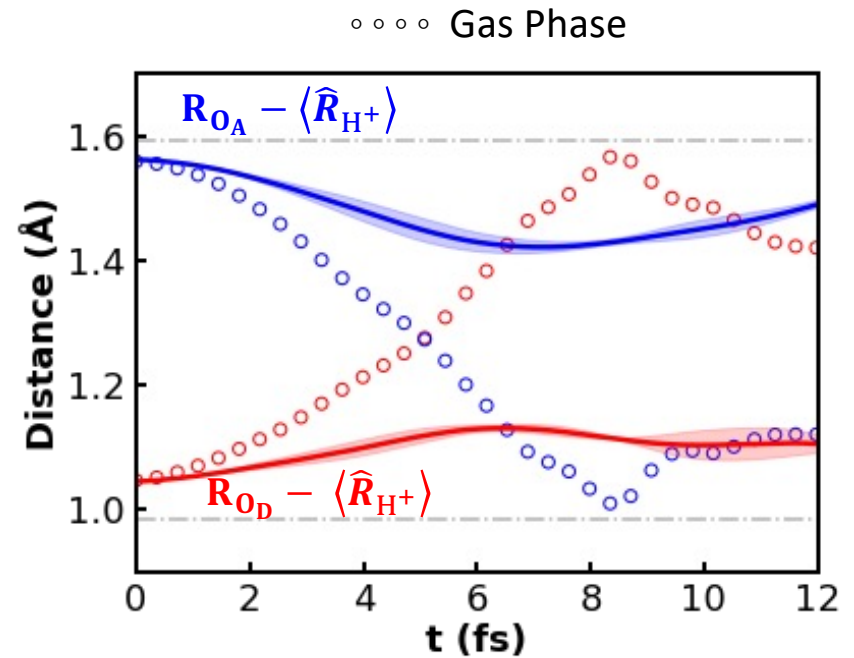
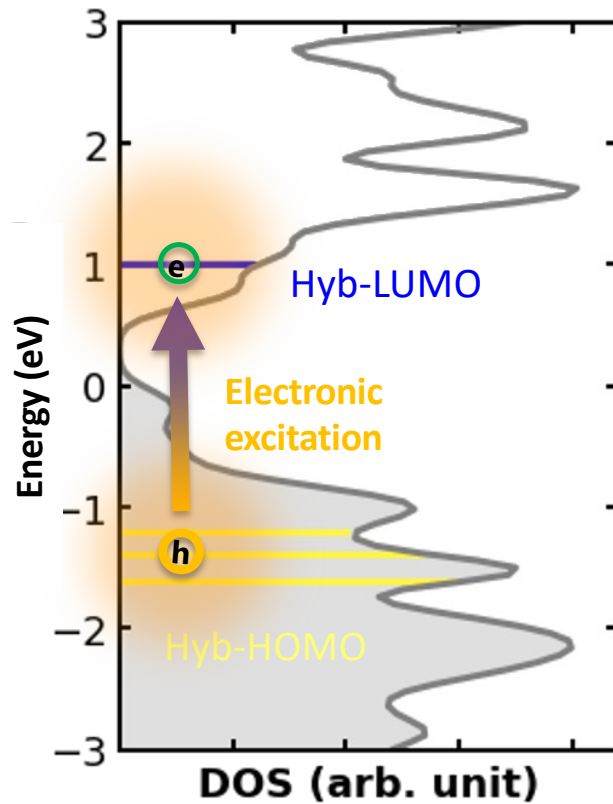
Electronic Excitation (HOMO \rightarrow LUMO)

oHBA attached on Semiconductor Surface

oHBA



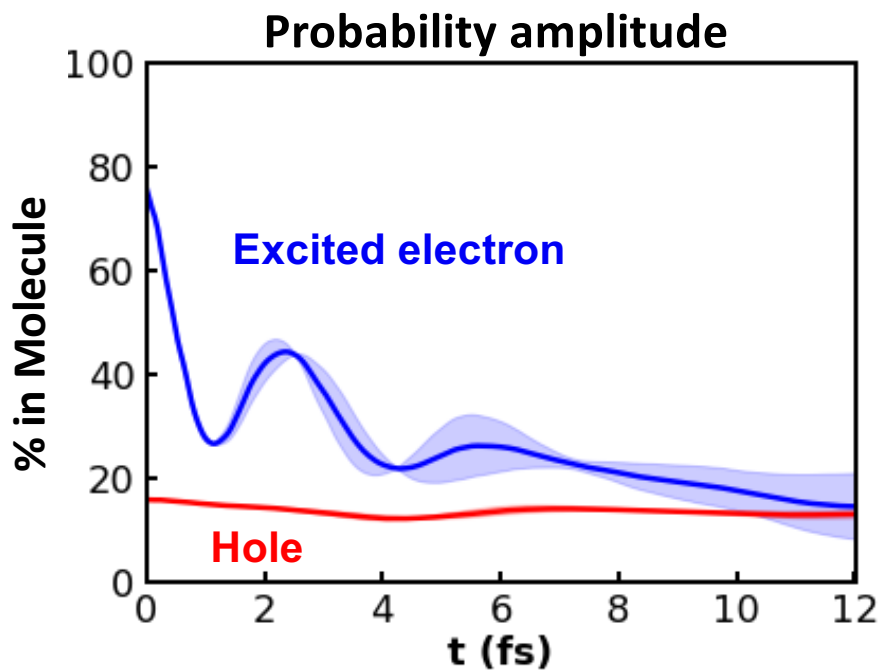
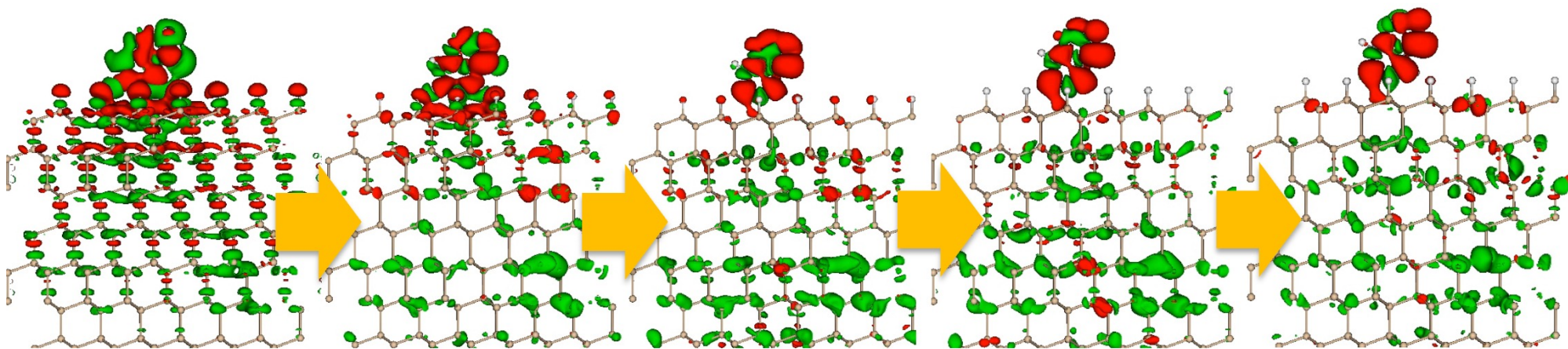
~3700 electrons.



H⁺ transfer does NOT take place!

Electron Density Changes at Interface

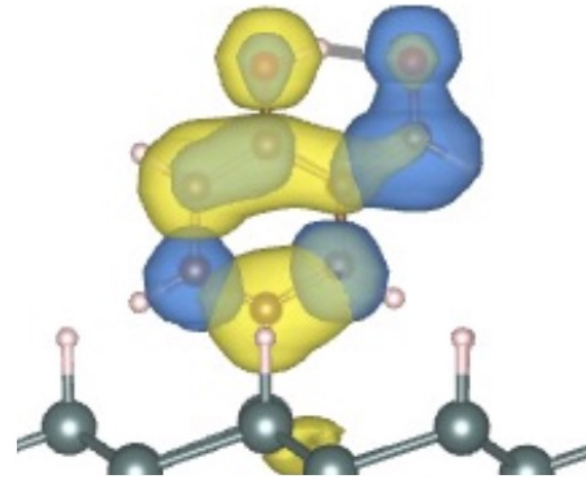
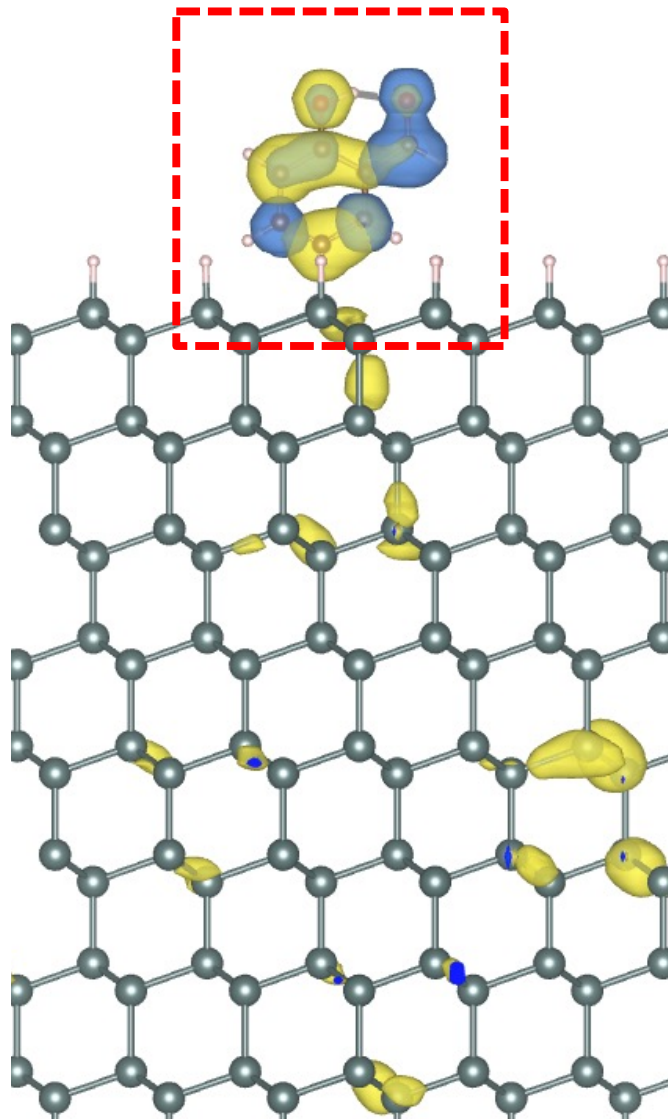
Electron density change : $\Delta\rho^e(\mathbf{r}, t)$ **Increases / Decreases**



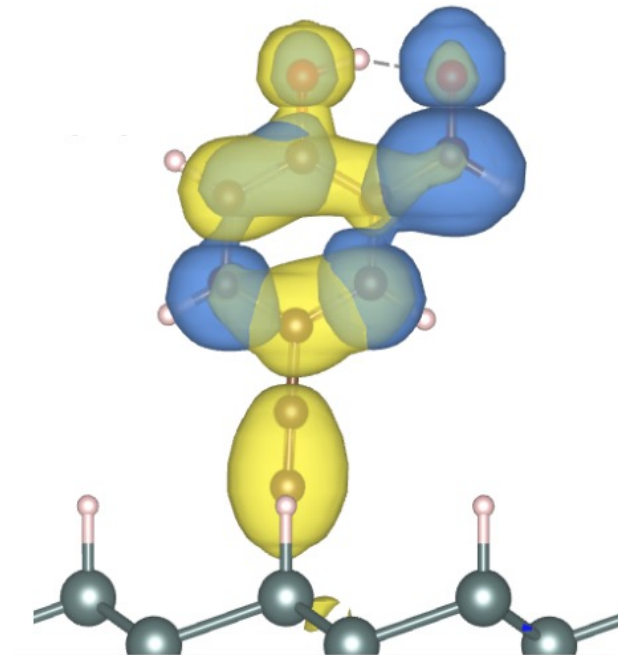
Ultrafast transfer of excited electron to semiconductor surface.

No driving force for the intra-molecular H^+ transfer.

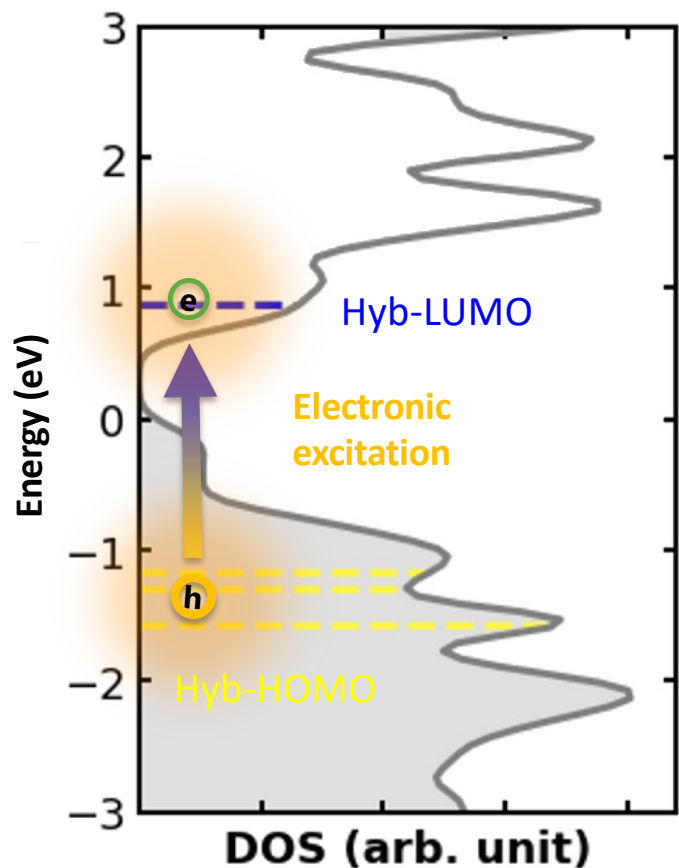
Role of Excited Electron Transfer



$-\text{C} \equiv \text{C}-$ **Linker**

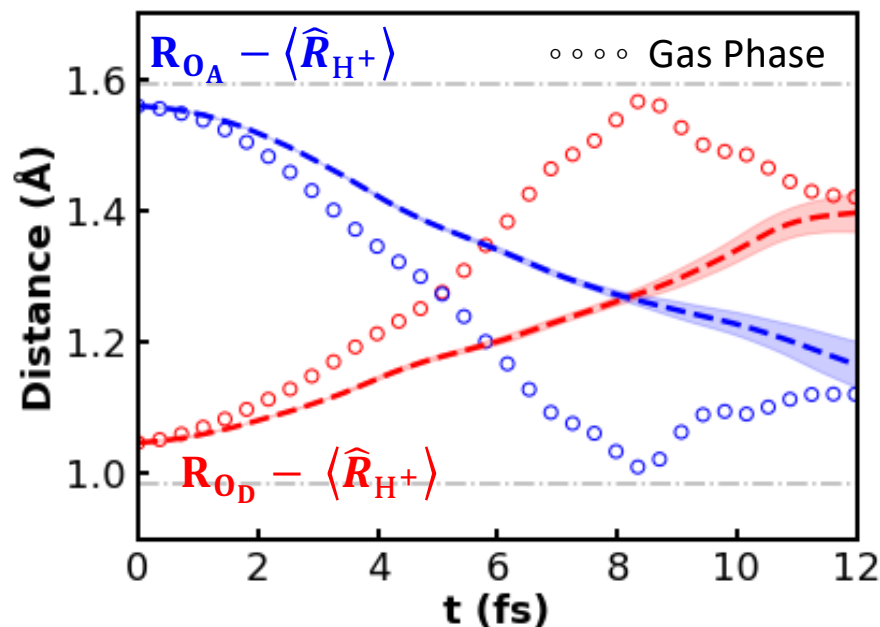
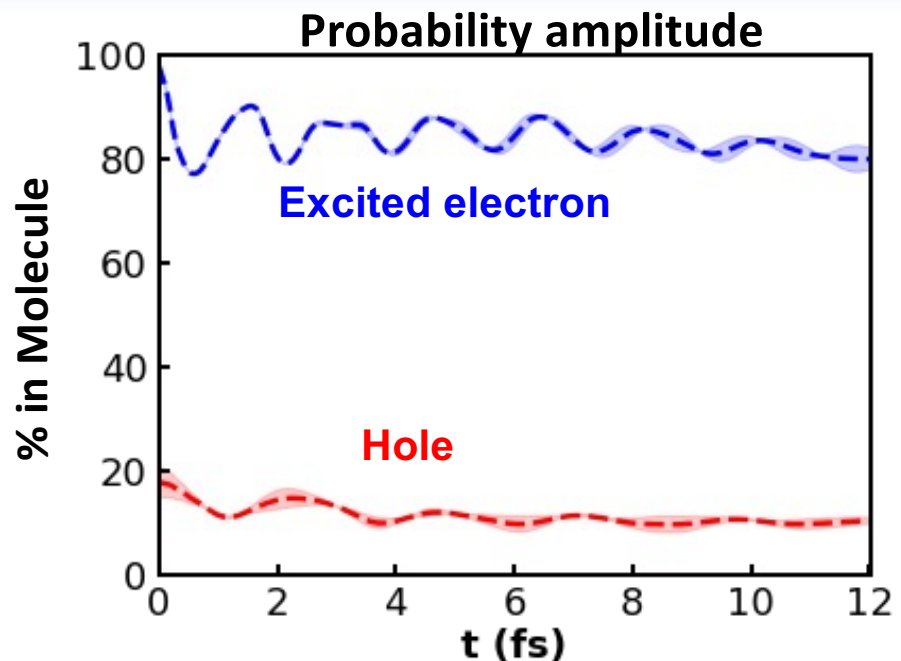


Controlling Electron Transfer w/ Linker Group

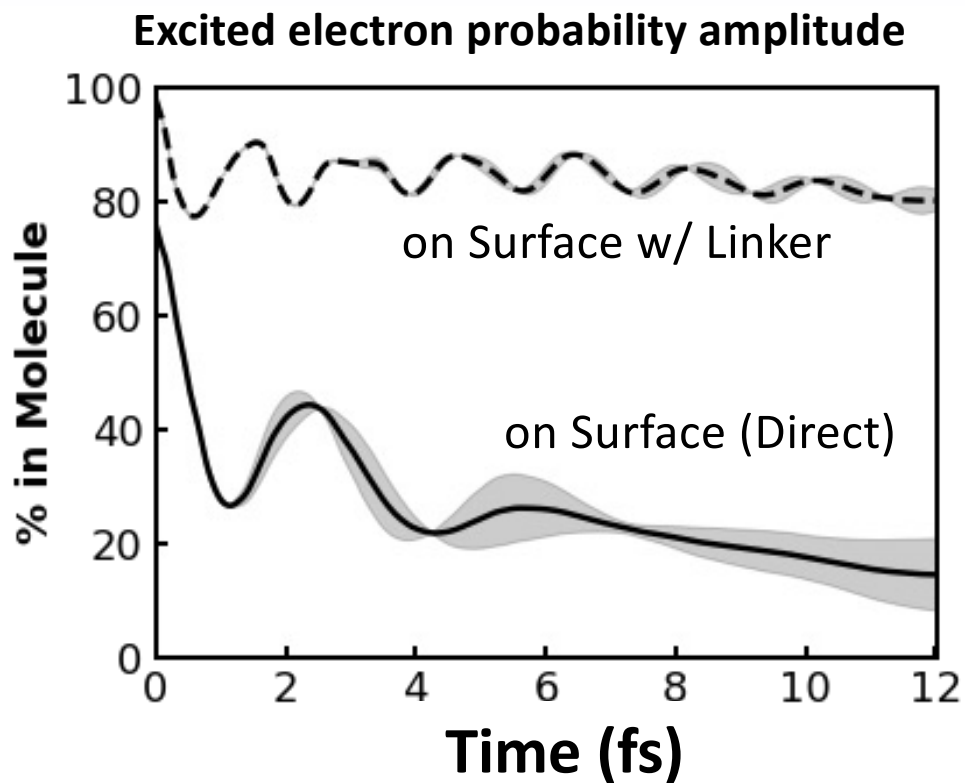
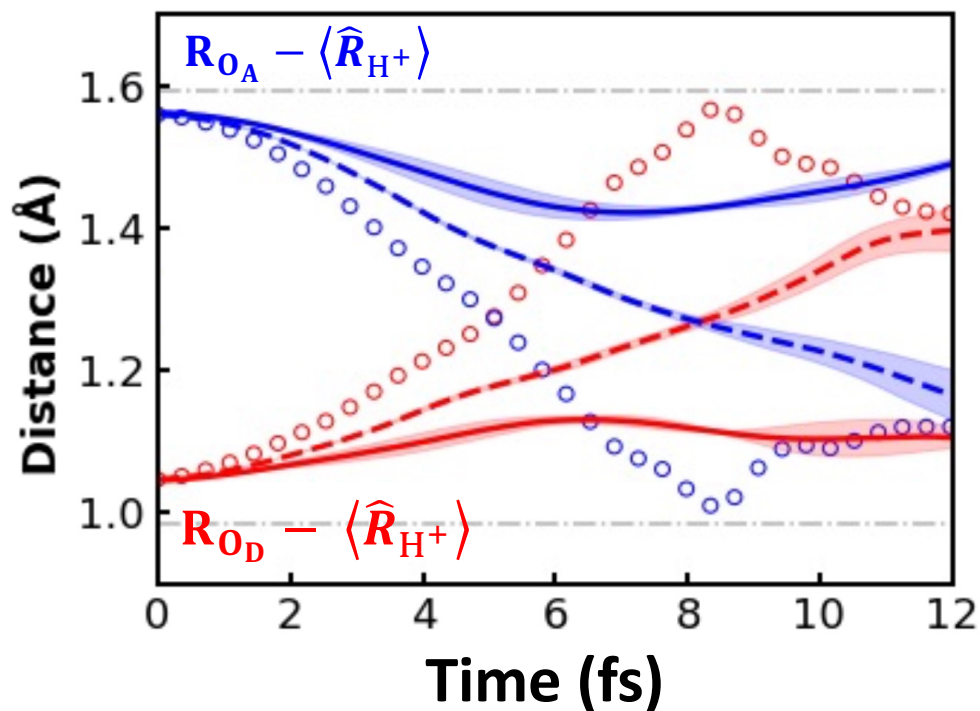


Excited electron transfer to the semiconductor is significantly slower with the linker group.

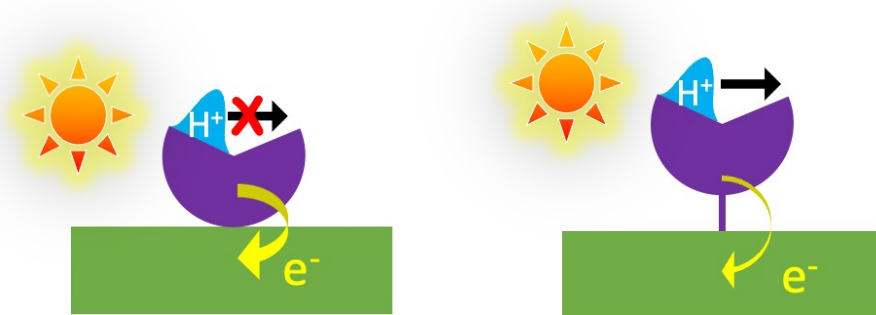
Excitation-induced H⁺ transfer takes place!



Dependence on the surface attachment



Surface attachment controls intra-molecular H^+ transfer via interfacial excited electron transfer.



Summary and References

NEO-DFT method for periodic systems.

Nuclear-Electronic Orbital Approach to Quantization of Protons in Periodic Electronic Structure Calculations

J. Xu, R. Zhou, Z. Tao, C. Malbon, V. Blum, S. Hammes-Schiffer, Y. Kanai

[J. Chem. Phys. 156, 224111 \(2022\)](#)

RT-TDDFT Implementation in all-electron periodic systems with NAO basis.

All-electron real-time and imaginary-time time-dependent density functional theory within a numeric atom-centered basis function framework

J. Hekele, Y. Yao, Y. Kanai, V. Blum, P. Kratzer

[J. Chem. Phys. 155, 154801 \(2021\)](#)

Periodic RT-NEO-TDDFT method for studying coupled electron-proton dynamics in heterogeneous systems

First-Principles Approach to Coupled Quantum Dynamics of Electrons and Protons in Heterogeneous Systems

J. Xu, R. Zhou, V. Blum, T. E. Li, S. Hammes-Schiffer, Y. Kanai

[Phys. Rev. Lett. 131, 238002 \(2023\)](#) [Editors' Suggestion](#)

Ehrenfest dynamics extension using Lagrangian formulation for traveling proton basis scheme

Lagrangian Formulation of RT-NEO-TDDFT Ehrenfest Dynamics (temp)

J. Xu, R. Zhou, T. E. Li, S. Hammes-Schiffer, Y. Kanai

[In preparation](#)



Thank you for listening!

