

VISTA Seminar, 2024

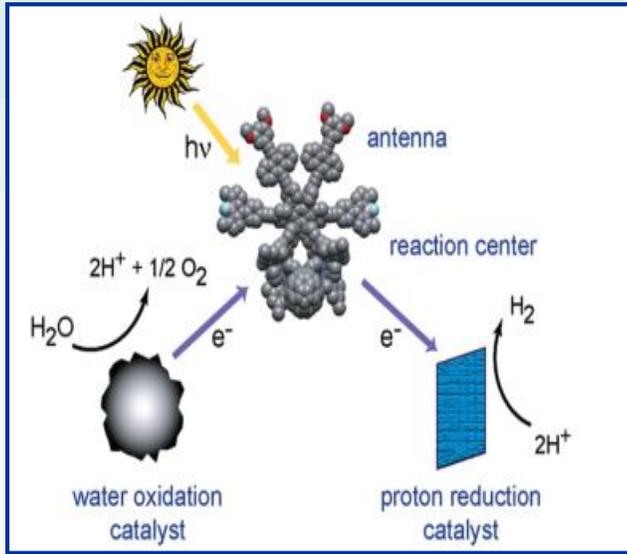
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# Nonadiabatic Dynamics and Machine Learning

Zhenggang Lan  
2024.02

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**South China Normal University**



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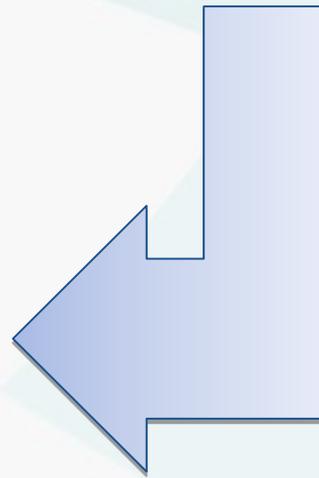
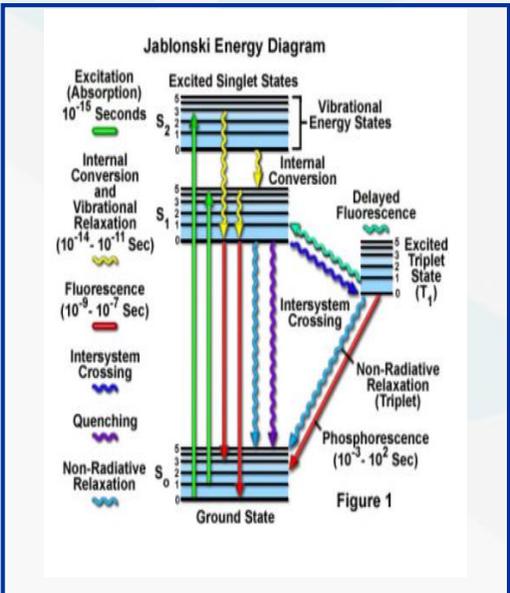
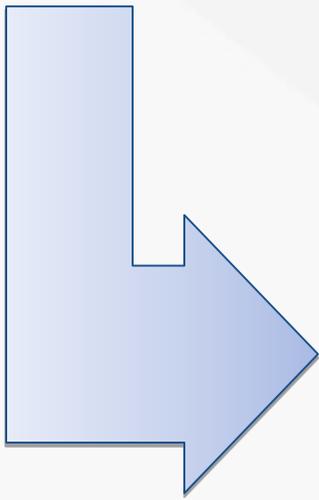


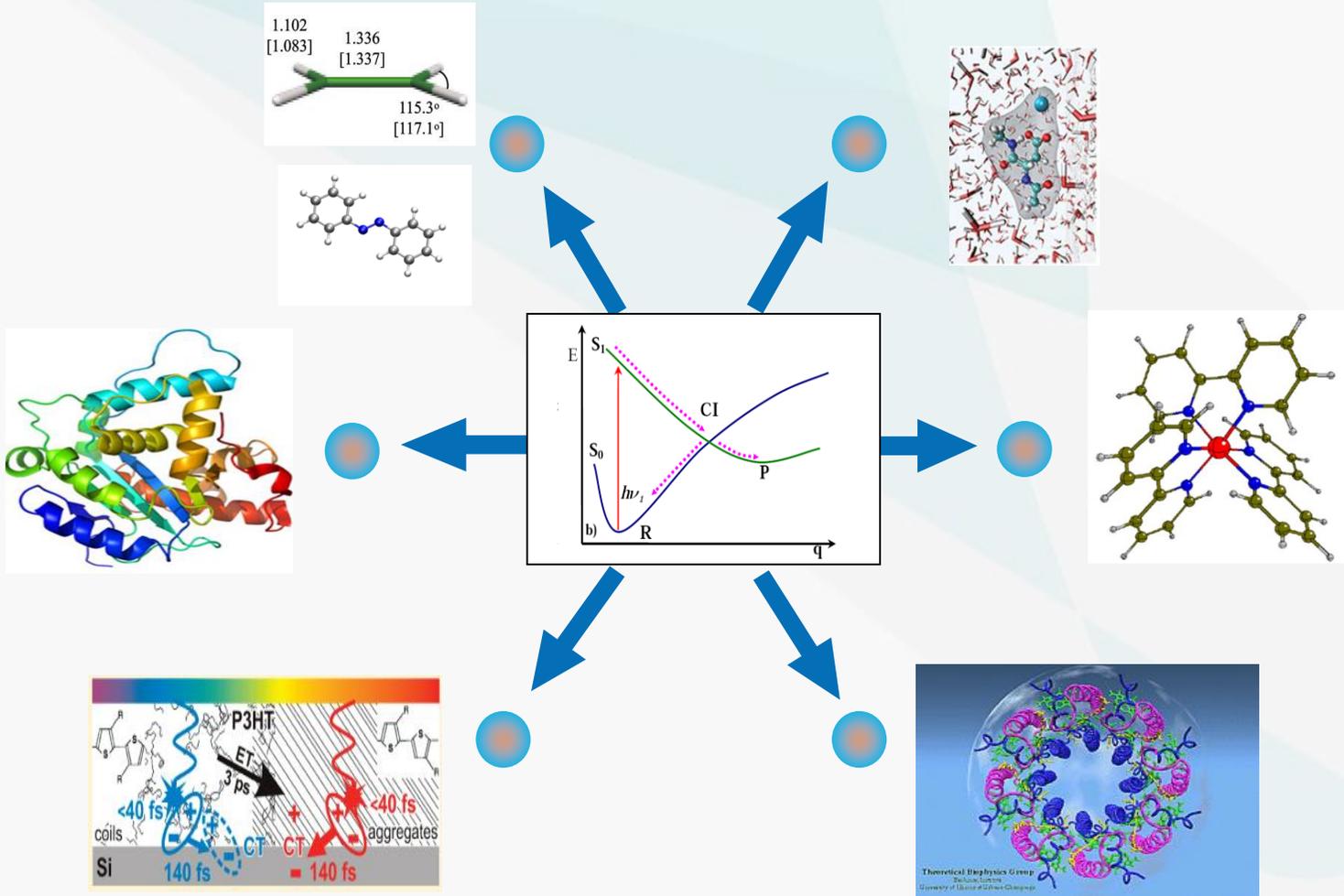
$$H\psi = E\psi, \quad i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}, \quad \frac{dq_i}{dt} = +\frac{\partial H}{\partial p_i}$$

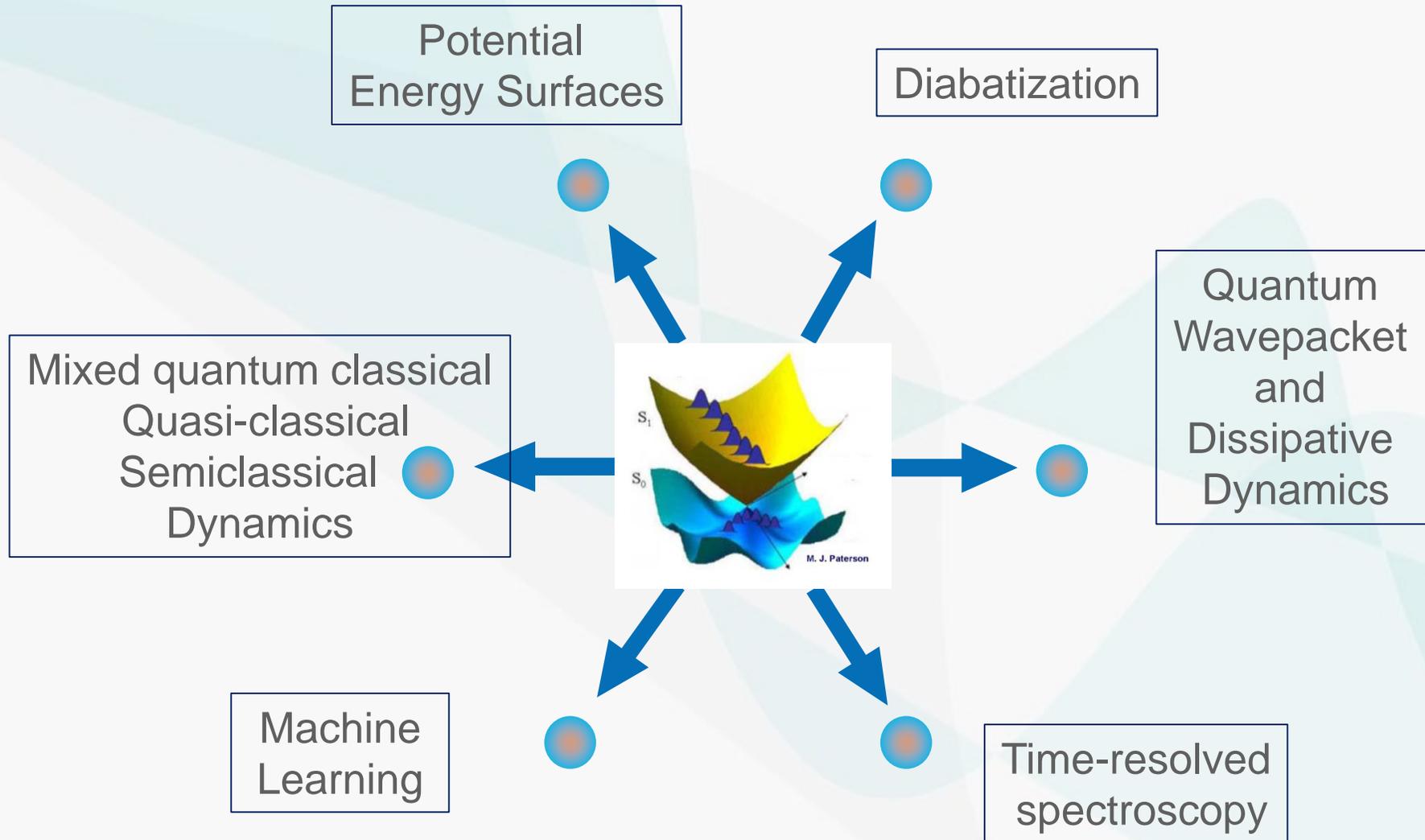
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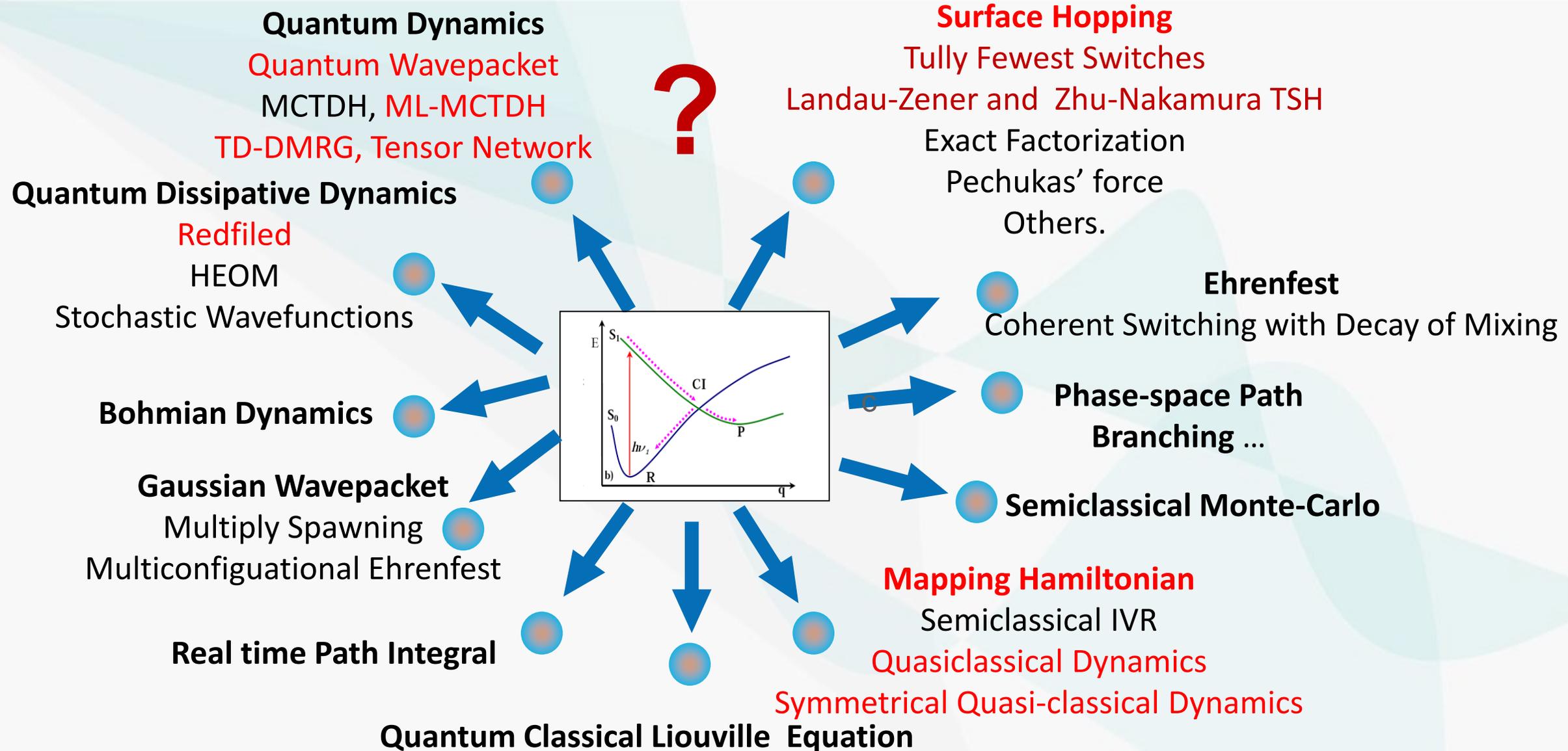


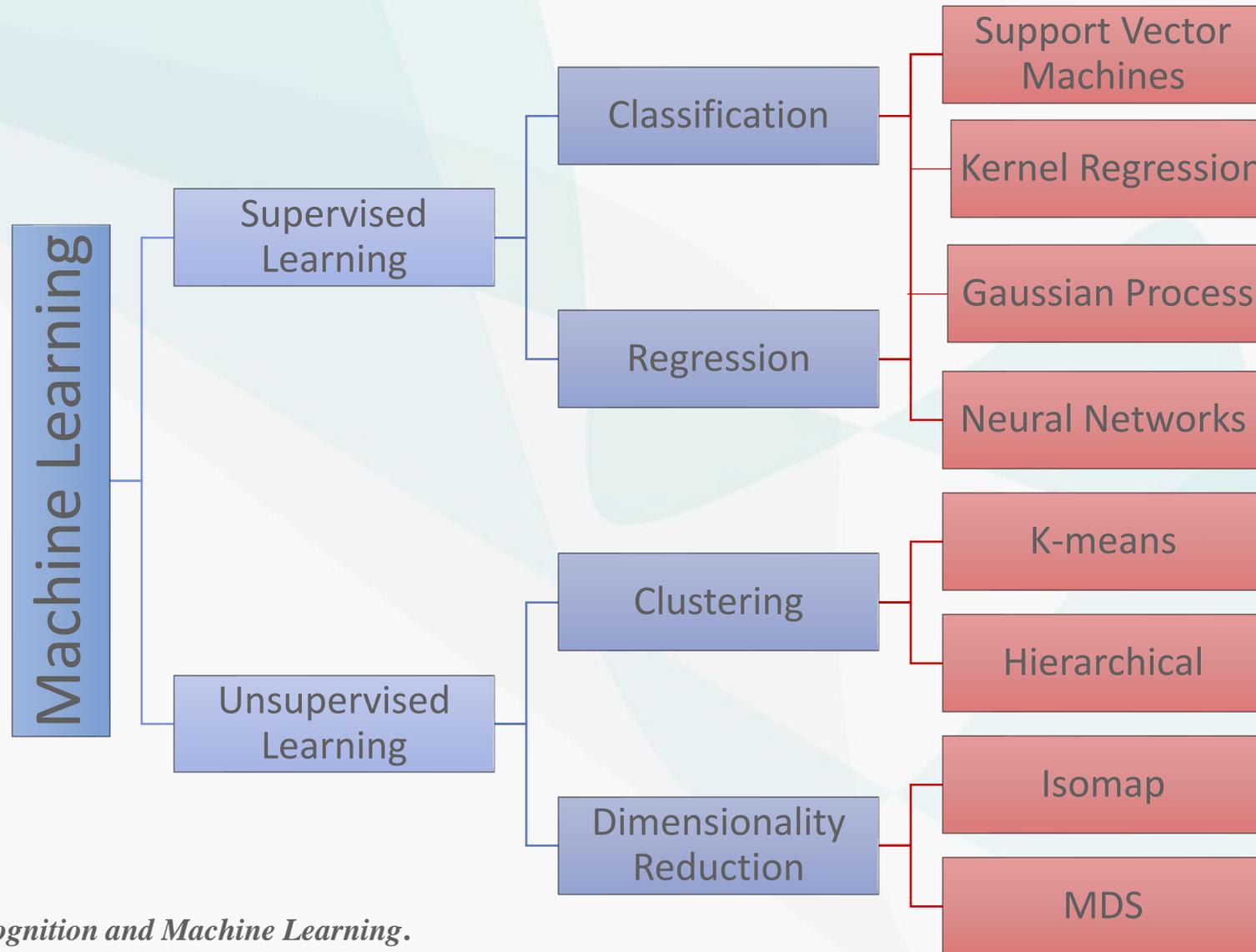


- Failure of Born-Oppenheimer approximation
- Complex excited-state electronic wavefunctions
- Complexity in realistic polyatomic systems
- Interpretations of experimental observations



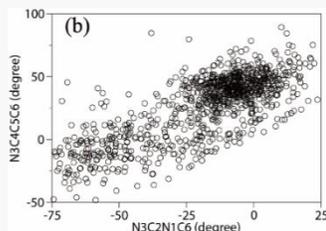
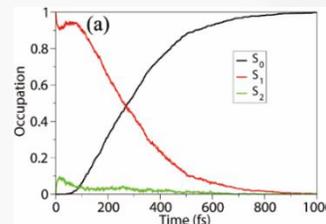
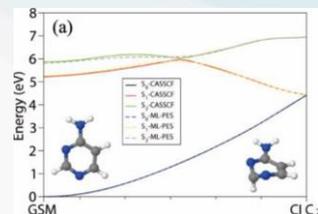
# Nonadiabatic Dynamics Methods





# Nonadiabatic Dynamics and Machine Learning

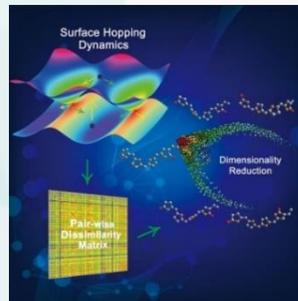
## PES Fitting



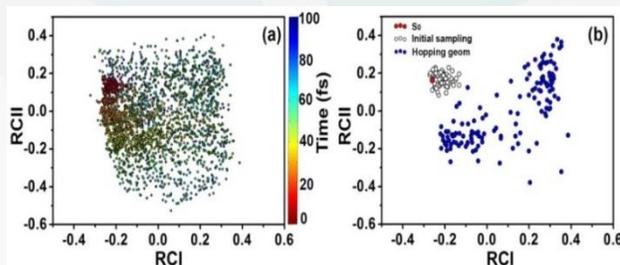
## Fast Dynamics

[D. Hu, Z. Lan\* et al., *J. Phys. Chem. Lett.*, 2018, 9, 2725]

## Result Analyses



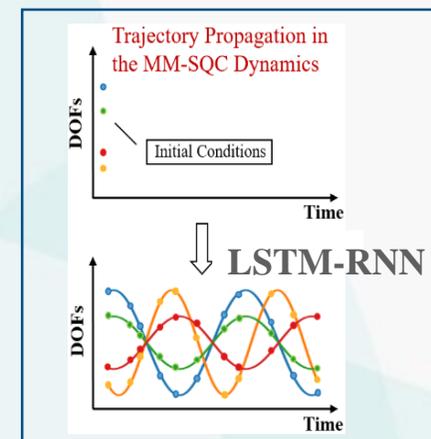
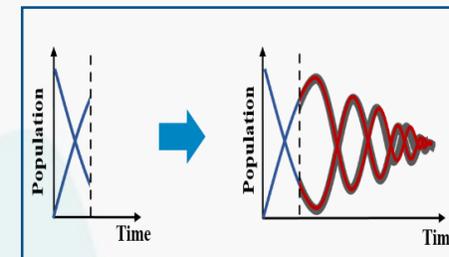
## Dimensionality Reduction



## Key Coordinates

[X. Li, Z. Lan\* et al., *J. Chem. Theory Comput.*, 2017, 13, 4611;  
X. Li, Z. Lan\* et al., *J. Chem. Phys.*, 2018, 149, 244104;  
J. Peng, Z. Lan\* et al., *J. Chem. Phys.*, 2021, 159, 094122;  
Y. Zhu, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2022, 24, 24362]

## Dynamics Evolution



## Direct Solutions

[K. Lin, F.G. Gu\*, Z. Lan\* et al., *J. Phys. Chem. Lett.*, 2021, 12, 10225;  
K. Lin, F.G. Gu\*, Z. Lan\* et al., *J. Phys. Chem. Lett.*, 2022, 13, 11678;  
K. Lin, F.G. Gu\*, Z. Lan\* et al., *J. Chem. Theory Comput.*, 2022, 18, 5837]

Open Quantum System  $H = H_S + H_B + H_{SB}$

## Environment

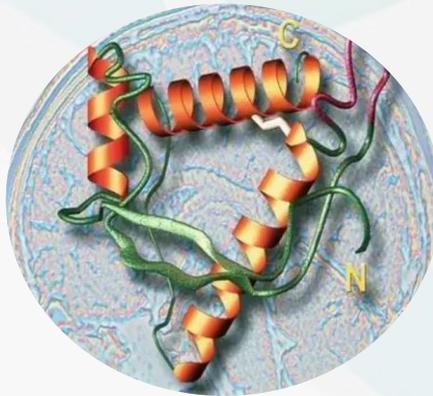
Hamiltonian  $H_B$   
System state  $\rho_B$

Interaction

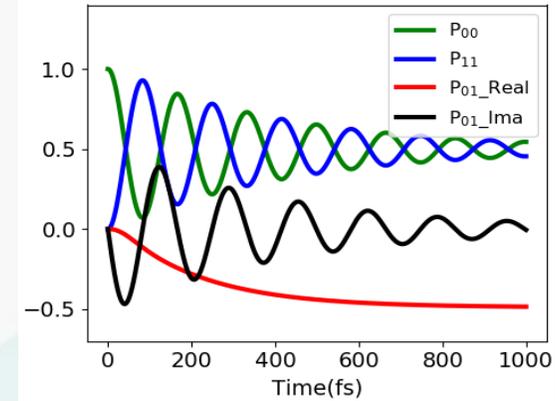
$H_{SB}$

## Open System

Hamiltonian  $H_S$   
System state  $\rho_S$



## System-plus-Bath Hamiltonian

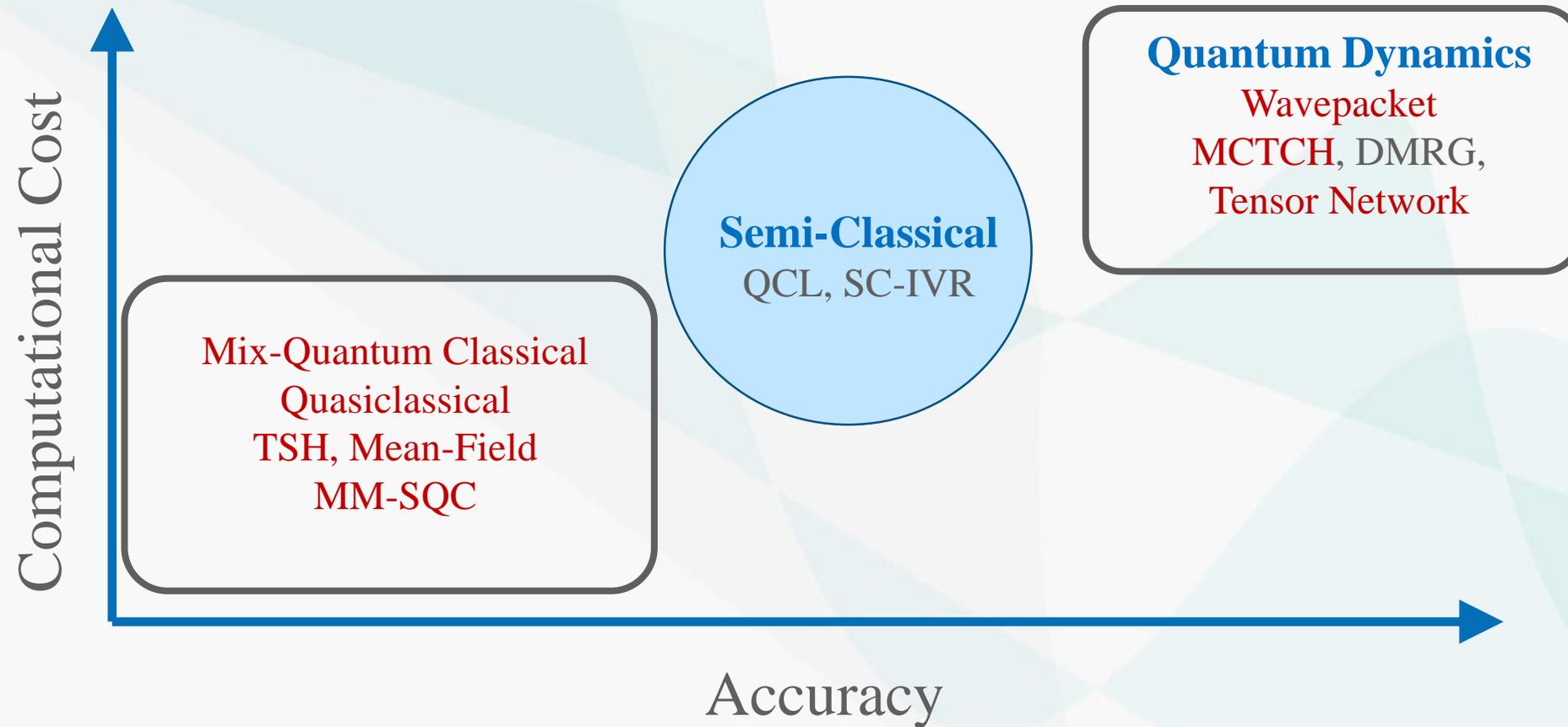


$$H = H_S + H_B + H_{SB}$$

$$H_S = \sum_{k=1}^2 |\varphi_k\rangle V_{kk} \langle \varphi_k| + \sum_{k \neq l} |\varphi_k\rangle V_{kl} \langle \varphi_l|,$$

$$H_B = \sum_{k=1}^2 \sum_j \frac{1}{2} \omega_{kj} (Q_{kj}^2 + P_{kj}^2),$$

$$H_{SB} = \sum_{k=1}^2 |\varphi_k\rangle \left( \sum_j \kappa_{kj} Q_{kj} \right) \langle \varphi_k|.$$

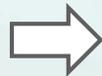


# ML-MCTDH and Tensor Network

Standard propagation method ( $f_{\max} \sim 10$ )

$$\Psi(Q_1, \dots, Q_f, t) = \sum_{j_1=1}^{N_1} \dots \sum_{j_f=1}^{N_f} C_{j_1 \dots j_f}(t) \prod_{\kappa=1}^f \chi_{j_\kappa}^{(\kappa)}(Q_\kappa)$$

$$i \frac{\partial}{\partial t} C_J = \sum_L H_{JL} C_L$$



Tensor-Train

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{P}_T \hat{H} |\Psi(t)\rangle \longrightarrow i\hbar \frac{d}{dt} |\Psi(t)\rangle = - \sum_{i=1}^{m-1} \hat{P}_i^L \otimes \hat{P}_{i+1}^R \hat{H} |\Psi(t)\rangle$$

$$|\Psi\rangle = \sum_{\{s_i\}} \mathbf{L}^{s_1} \dots \mathbf{L}^{s_i} \dots \mathbf{L}^{s_m} |s_1 \dots s_i \dots s_m\rangle = \sum_{\{s_i\}} \mathbf{R}^{s_1} \dots \mathbf{R}^{s_i} \dots \mathbf{R}^{s_m} |s_1 \dots s_i \dots s_m\rangle$$

$$= \sum_{\{s_i\}} \mathbf{L}^{s_1} \dots \mathbf{L}^{s_{i-1}} \mathbf{M}^{s_i} \mathbf{R}^{s_{i-1}} \dots \mathbf{R}^{s_m} |s_1 \dots s_i \dots s_m\rangle$$

$$= \sum_{\{\alpha_{i-1}, s_i, \alpha_i\}} [\mathbf{M}^{s_i}]_{\alpha_{i-1}, \alpha_i} |\Psi_{L, \alpha_{i-1}}^{[1:i-1]}\rangle |s_i\rangle |\Psi_{R, \alpha_i}^{[i+1:m]}\rangle$$

$$\hat{O} = \sum_{\{s_i\}, \{s'_i\}, \{\beta_i\}} \mathbf{W}_{\beta_0, \beta_1}^{s_1, s'_1} \dots \mathbf{W}_{\beta_{i-1}, \beta_i}^{s_i, s'_i} \dots \mathbf{W}_{\beta_{m-1}, \beta_m}^{s_m, s'_m} |s_1 \dots s_i \dots s_m\rangle \langle s'_1 \dots s'_i \dots s'_m|$$

$$\hat{P}_T = \sum_{i=1}^m \hat{P}_{i-1}^L \otimes \hat{I}_i \otimes \hat{P}_{i+1}^R - \sum_{i=1}^{m-1} \hat{P}_i^L \otimes \hat{P}_{i+1}^R$$

$$\hat{P}_i^L = \sum_{\alpha_i} |\Psi_{L, \alpha_i}^{[1:i]}\rangle \langle \Psi_{L, \alpha_i}^{[1:i]}|$$

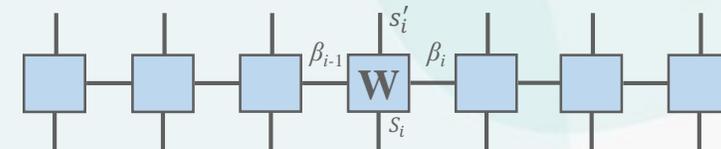
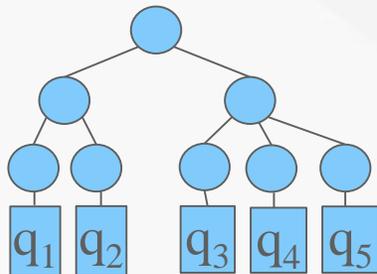
$$\hat{P}_i^R = \sum_{\alpha_i} |\Psi_{R, \alpha_{i-1}}^{[i:m]}\rangle \langle \Psi_{R, \alpha_{i-1}}^{[i:m]}|$$

ML-MCTDH ( $f_{\max} \sim 1500$ )

$$\varphi_m^{z-1}(Q_{\kappa_{i-1}}^{z-1}, t) = \sum_{j_1=1}^{n_{j_1}} \dots \sum_{j_{p_{\kappa_i}}=1}^{n_{j_{p_{\kappa_i}}}} A_{j_1 \dots j_{p_{\kappa_i}}}^z(t) \prod_{\kappa_i=1}^{p_{\kappa_i}} \varphi_{j_{\kappa_i}}^{z, \kappa_i}(Q_{\kappa_i}^z, t)$$

$$i \dot{A}_J^l = \sum_L \langle \Phi_J^l | H | \Phi_L^l \rangle A_L^l$$

$$i \dot{\varphi}_n^{z, \kappa_i} = (1 - \hat{P}_{\kappa_i}^z) \sum (\rho^{z, \kappa_i})_{nj}^{-1} \langle H \rangle_{jm}^{z, \kappa_i} \varphi_m^{z, \kappa_i}$$



[*J. Chem. Phys.*, 128:164116 (2008)]

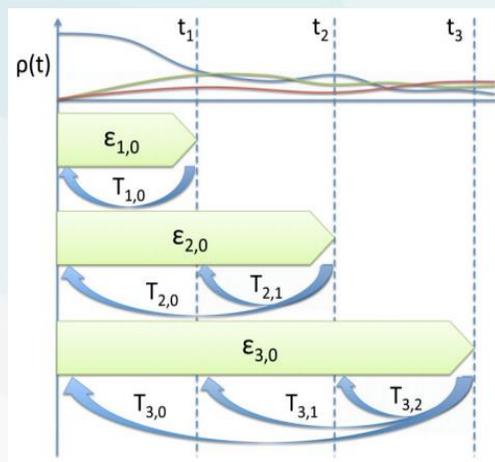
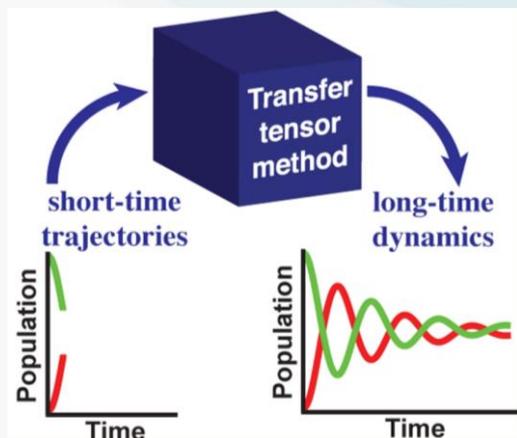
[*J. Chem. Phys.*, 119:1289 (2003)]

[*Phys. Rep.*, 324:100-105 (2000)]

[*J. Phys. Chem. A*, 119:7951-7965 (2015)]

[*Ann. Phys.* 326:96 (2011)]

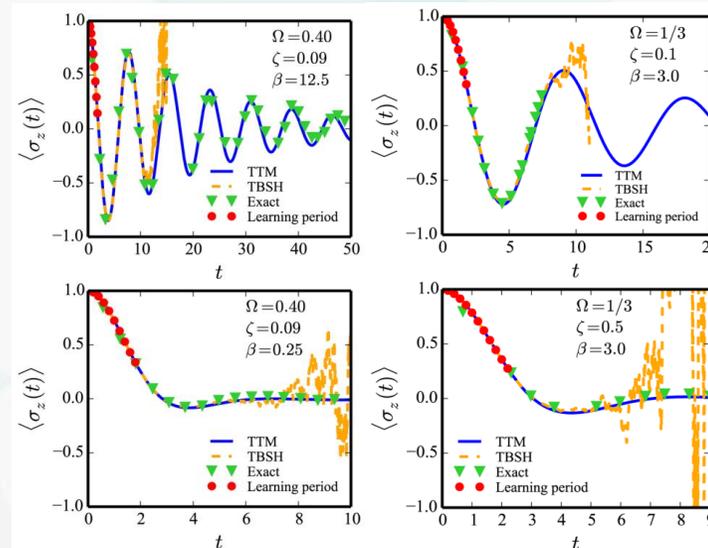
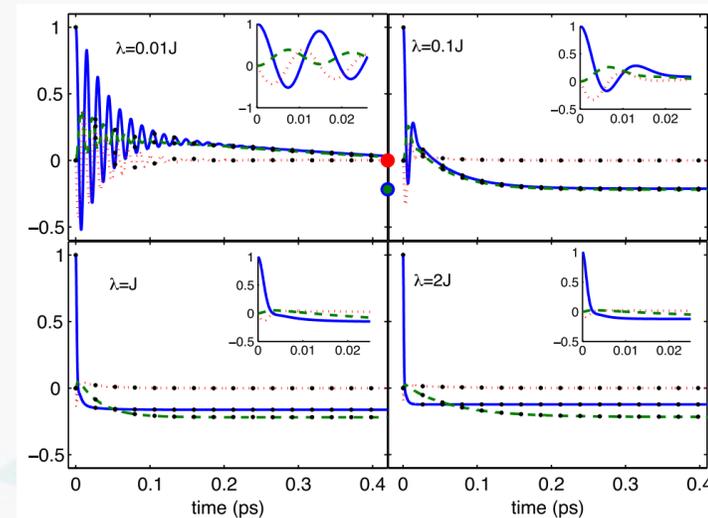
[*Phys. Rev. B* 94:165116 (2016)]



$$T_{n,0} = \mathcal{E}_n - \sum_{m=1}^{n-1} T_{n,m} \mathcal{E}_m,$$

$$\rho(t_n) = \sum_{k=0}^{n-1} T_{n,k} \rho(t_k)$$

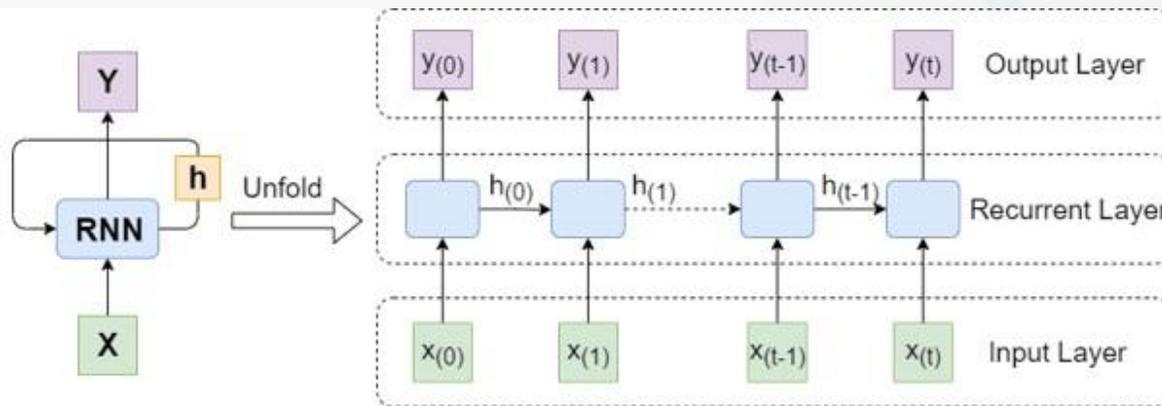
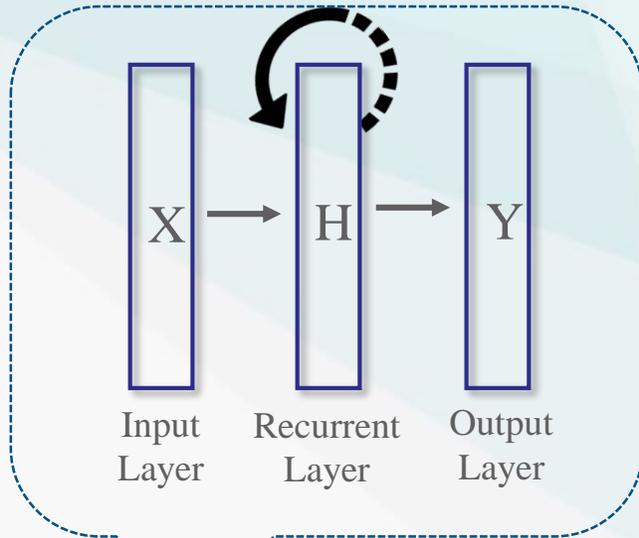
$$\rho(t_m) = (T_1 T_2 \dots T_K) \begin{pmatrix} \rho(t_{m-1}) \\ \rho(t_{m-2}) \\ \vdots \\ \rho(t_{m-K}) \end{pmatrix}$$



[Kananenka, A. A.; Hsieh, C.-Y.; Cao, J.; Geva, E. *J. Phys. Chem. Lett.*, 7, 4809-4814 (2016)]

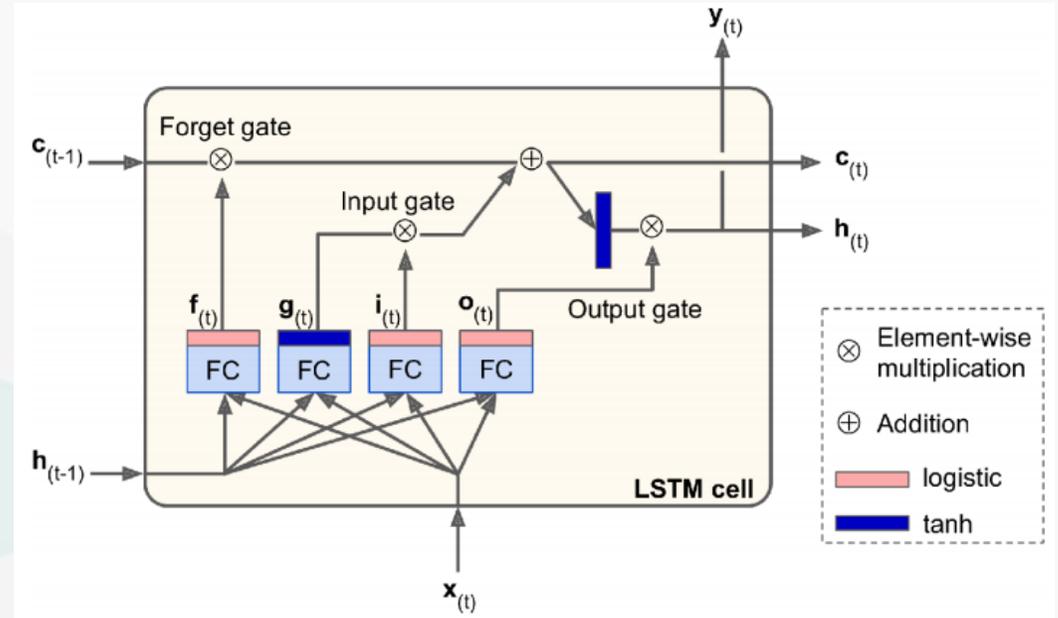
[Cerrillo, J.; Cao, J. *Phy. Rev. Lett.*, 112, 110401 (2014)]

- The Simple RNN



[Goodfellow, I.; Bengio, Y.; Courville, A. *Deep Learning*. MIT press: 2016.]

- The LSTM Cell



$$i_{(t)} = \sigma(W_{xi}^T x_{(t)} + W_{hi}^T h_{(t-1)} + b_i),$$

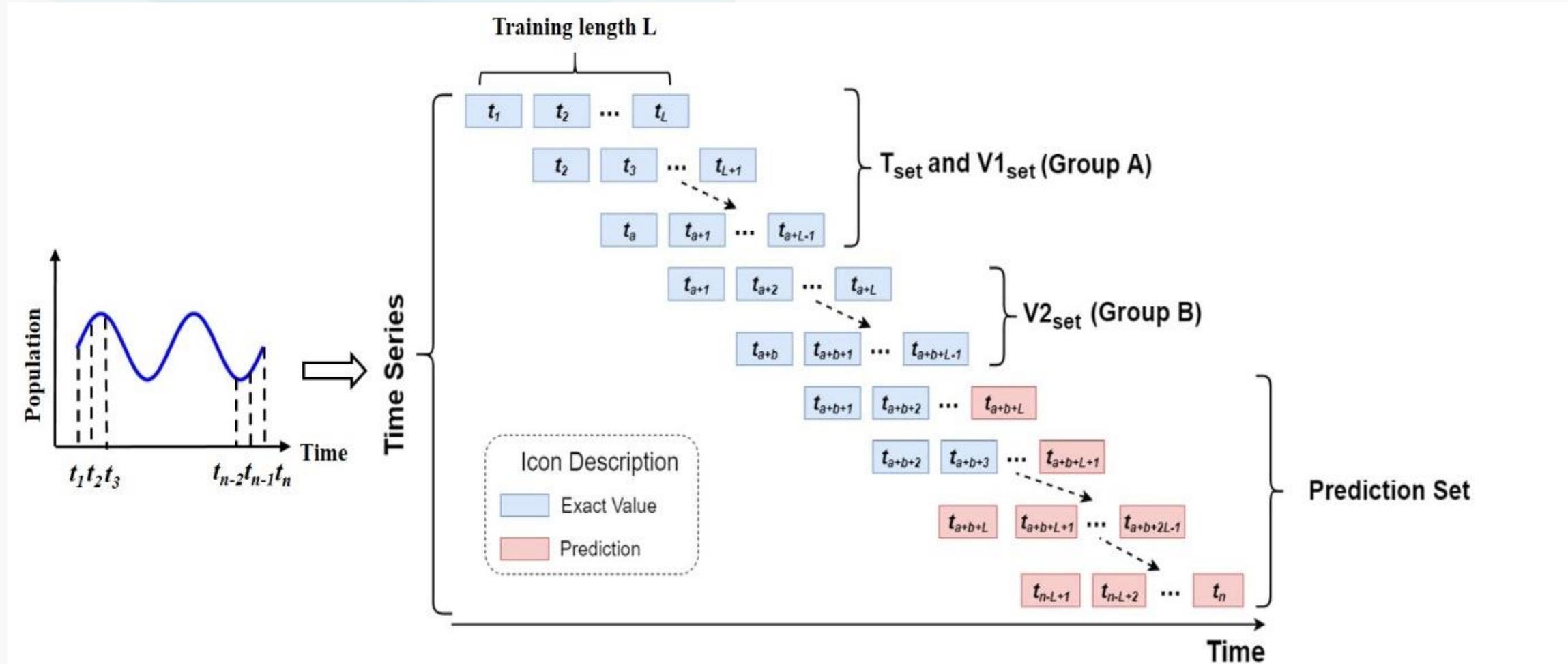
$$f_{(t)} = \sigma(W_{xf}^T x_{(t)} + W_{hf}^T h_{(t-1)} + b_f),$$

$$o_{(t)} = \sigma(W_{xo}^T x_{(t)} + W_{ho}^T h_{(t-1)} + b_o),$$

$$g_{(t)} = \tanh(W_{xg}^T x_{(t)} + W_{hg}^T h_{(t-1)} + b_g),$$

$$c_{(t)} = f_{(t)} \cdot c_{(t-1)} + i_{(t)} \cdot g_{(t)},$$

$$y_{(t)} = h_{(t)} = o_{(t)} \cdot \tanh(c_{(t)}).$$



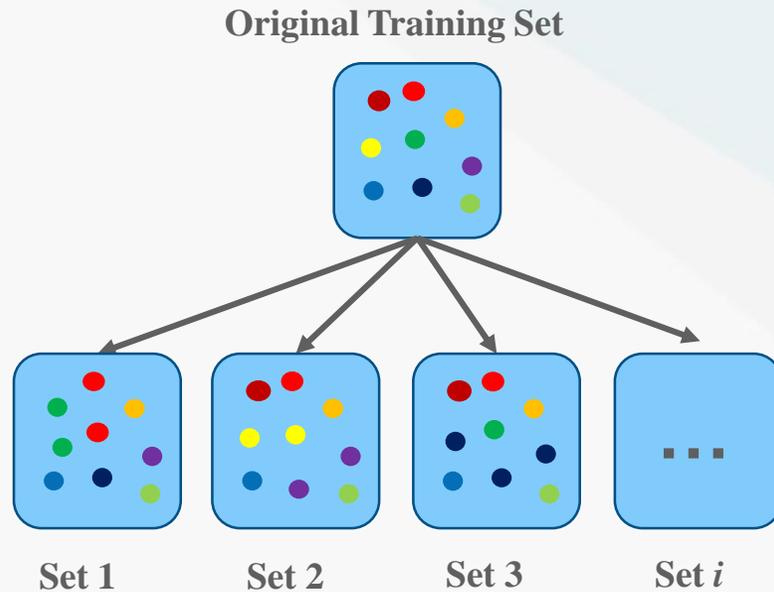
The estimation of the uncertainty is un-avoidable in all ML models

Two Uncertainties:

Model Misspecification

Model Uncertainty

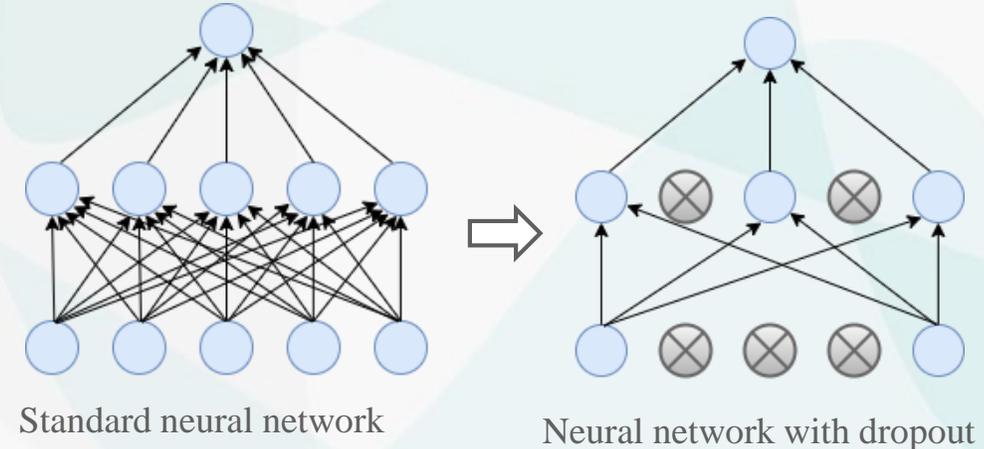
- **Bootstrap Resampling Method**



[Zhu, L.; Laptev, N. **IEEE**: 103-110(2017)]

[Bühlmann, P. *Stat. Sci.* 17, 52-72(2002)]

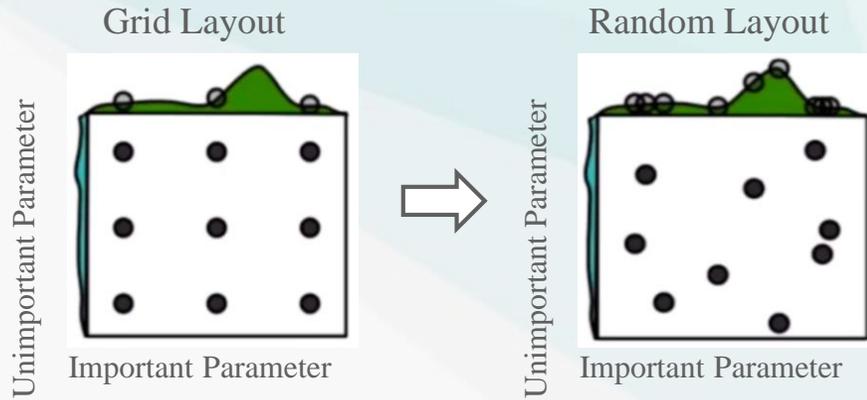
- **Monte-Carlo Dropout Method**



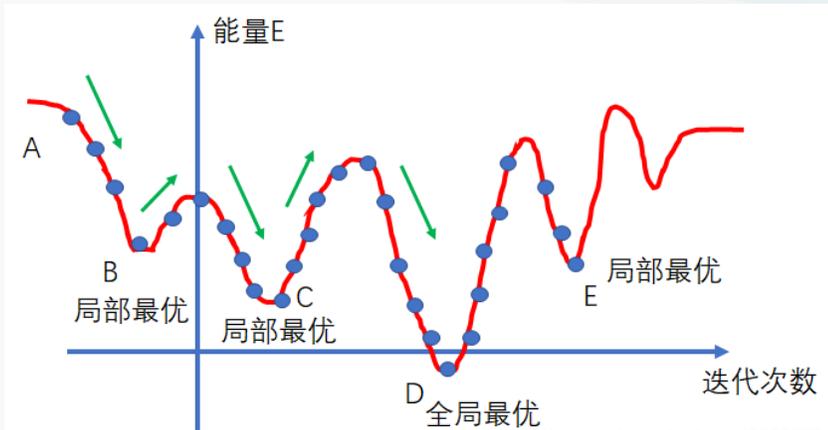
[Gal, Y.; Ghahramani, Z. **PMLR**: 48, 1050-1059(2016)]

[Gal, Y.; Ghahramani, Z. *Advances in neural information processing systems*, 29(2016)]

- **Random Search**



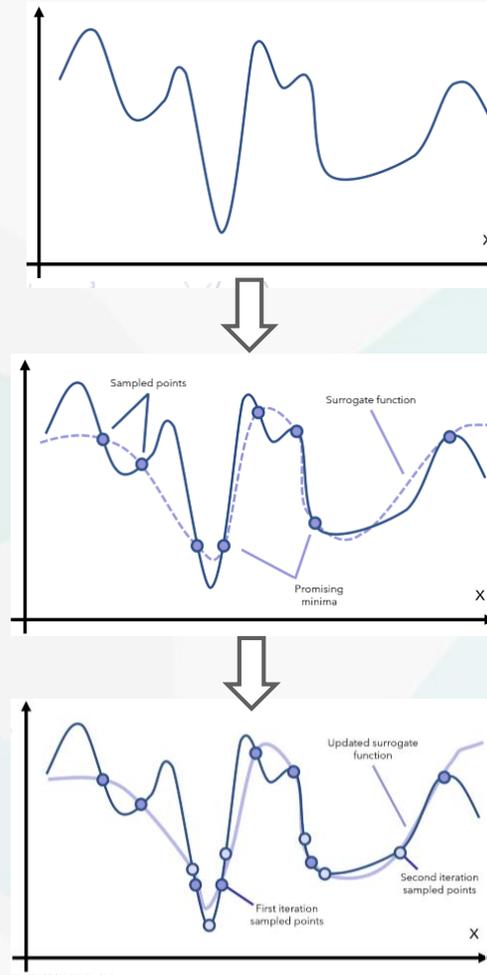
- **Simulated Annealing**



$$p = \begin{cases} \exp\left(-\frac{f(x_{t+1}) - f(x_t)}{T}\right) & \text{if } f(x_{t+1}) - f(x_t) \geq 0 \\ 1 & \text{if } f(x_{t+1}) - f(x_t) < 0 \end{cases}$$

[Yang, L.; Shami, A. *Neurocomputing*, 415, 295-316(2020)]

- **Bayesian Optimization with TPE**



Hyperparameters

$$\{x_i, y_i\}$$

Loss function

Parzen window

$$p(x | y) = \begin{cases} l(x), & \text{if } y < y^* \\ g(x), & \text{if } y \geq y^* \end{cases}$$

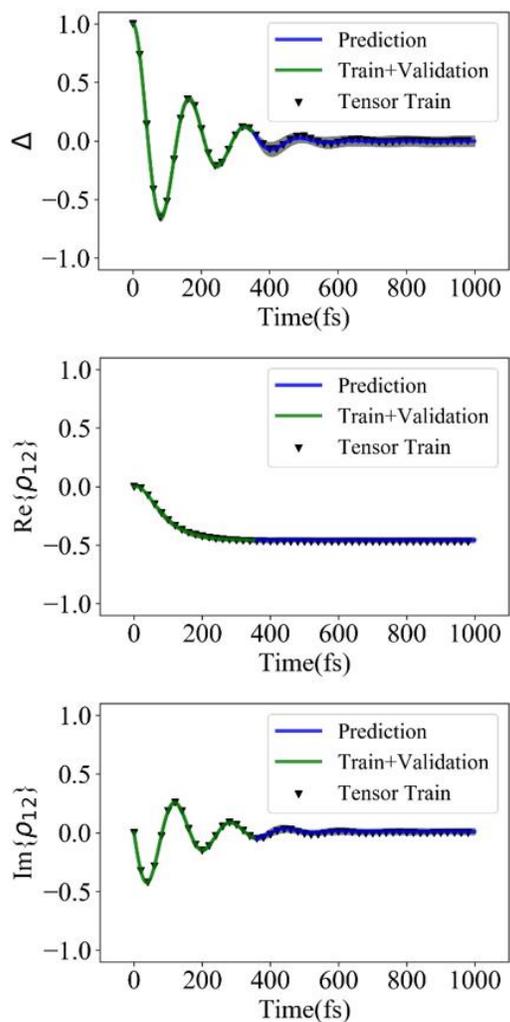
Maximizing EI

$$EI_{y^*}(x) \propto \left(\gamma + \frac{g(x)}{l(x)}(1-\gamma)\right)^{-1},$$

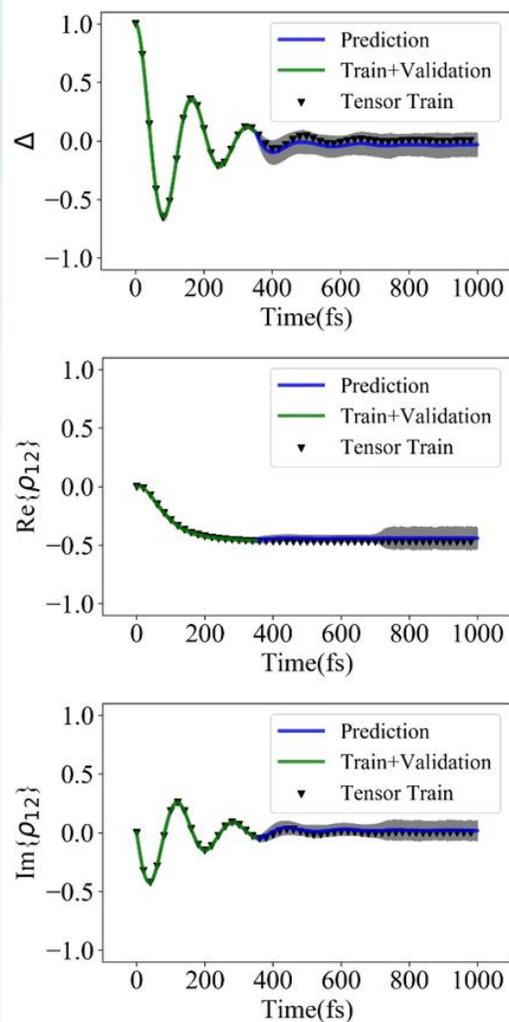
$$\gamma = p(y < y^*)$$

$$p(x) = \gamma l(x) + (1-\gamma)g(x).$$

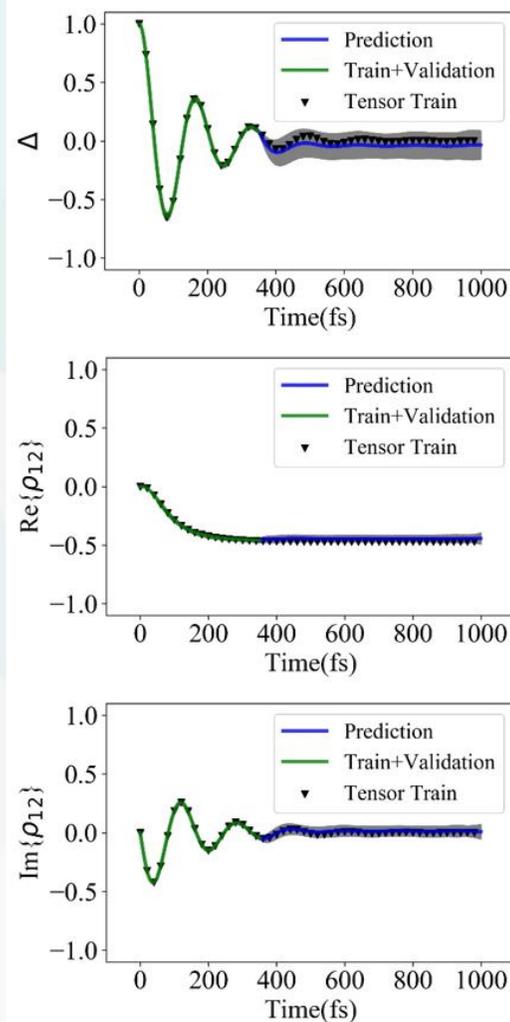
[Kirkpatrick, S.; Gelatt Jr, C. D.; Vecchi, M. P. *Science*, 220, 671-680(1983)]



(a) Simulated Annealing + Bootstrap + MC dropout



(b) Bayesian Optimization + Bootstrap + MC dropout



(c) Random Search + Bootstrap + MC dropout

## Model I (symmetric)

$$V_{12} = 0.0124 \text{ eV}$$

$$\omega_c = 200 \text{ cm}^{-1}$$

$$\lambda = 64 \text{ cm}^{-1}$$

Recommended  
Choice



Simulated Annealing

+

Bootstrap

+

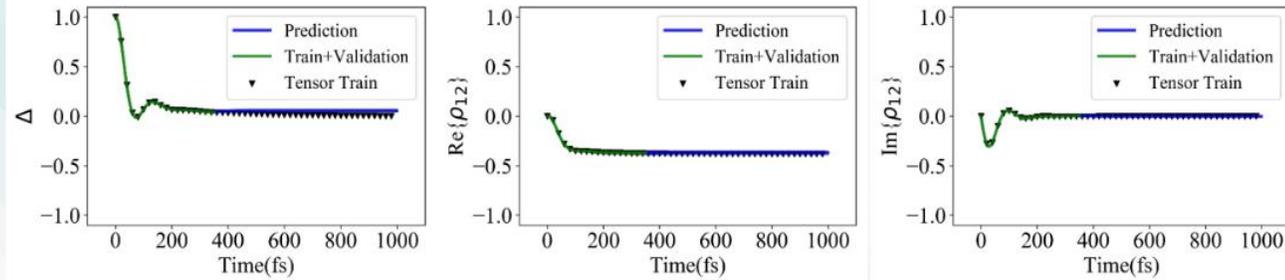
MC dropout

[Lin, K.; Peng, J.; Gu, F. L.; Lan, Z. *J. Phys. Chem. Lett.*, 12(41), 10225-10234 (2021)]

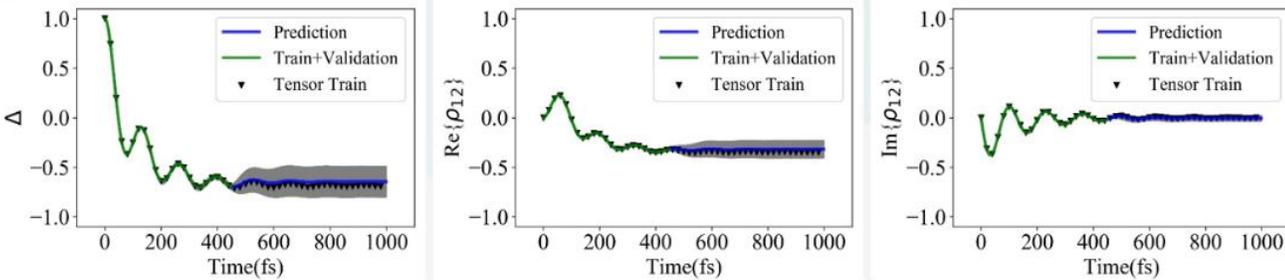
[Lin, K.; Peng, J.; Xu, C.; Gu, F. L.; Lan, Z. *J. Chem. Theory Comput.*, 18(10), 5837-5855 (2022)]

# Simulation of Open Quantum Dynamics with Uncertainty Analysis 17

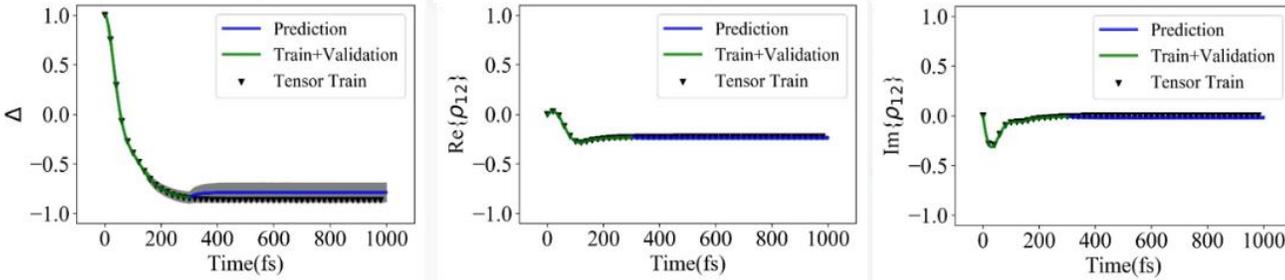
(a) Model II  
(asymmetric)



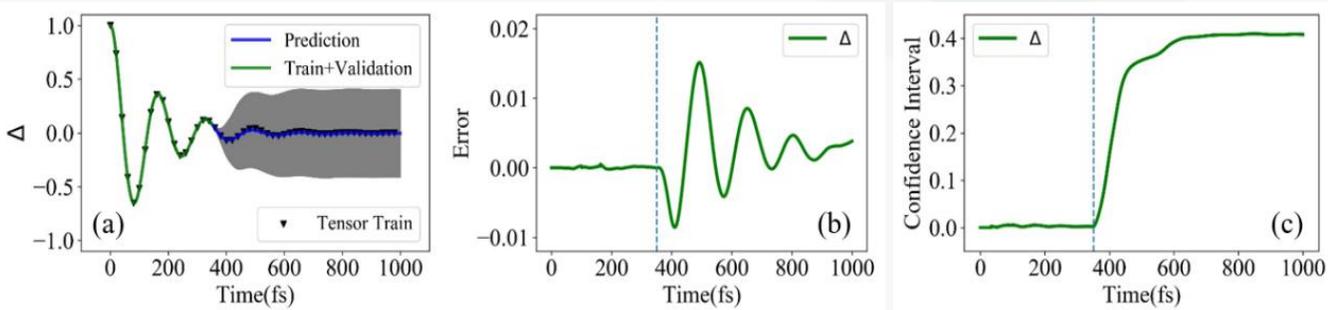
(b) Model III  
(asymmetric)



(c) Model IV  
(asymmetric)



**Model I**  
(symmetric)  
Only Electronic  
Populations

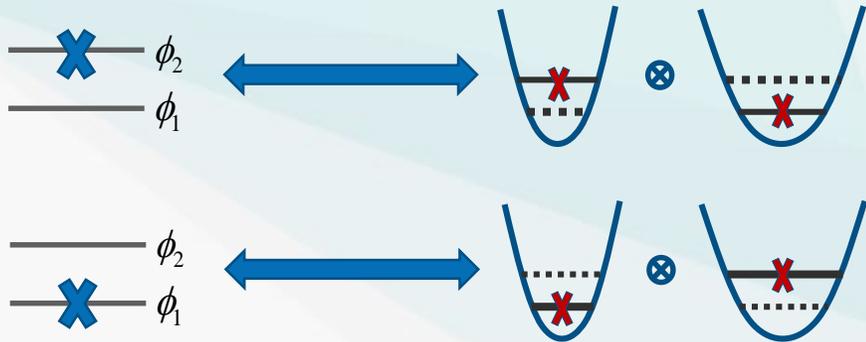


The excellent prediction results of symmetric and asymmetric site-exciton models.

[Lin, K.; Peng, J.; Gu, F. L.; Lan, Z.  
*J. Phys. Chem. Lett.*, 12(41), 10225-10234 (2021)]

[Lin, K.; Peng, J.; Xu, C.; Gu, F. L.; Lan, Z.  
*J. Chem. Theory Comput.*, 18(10), 5837-5855 (2022)]

The off-diagonal elements of the density matrix play a very important role.



$$\hat{H} = \sum_{n,m} \hat{h}_{nm} |\phi_n\rangle \langle \phi_m|$$

$$|\phi_n\rangle \langle \phi_m| \mapsto a_n^\dagger a_m,$$

$$|\phi_n\rangle \mapsto |0_1 \dots 1_n \dots 0_N\rangle.$$

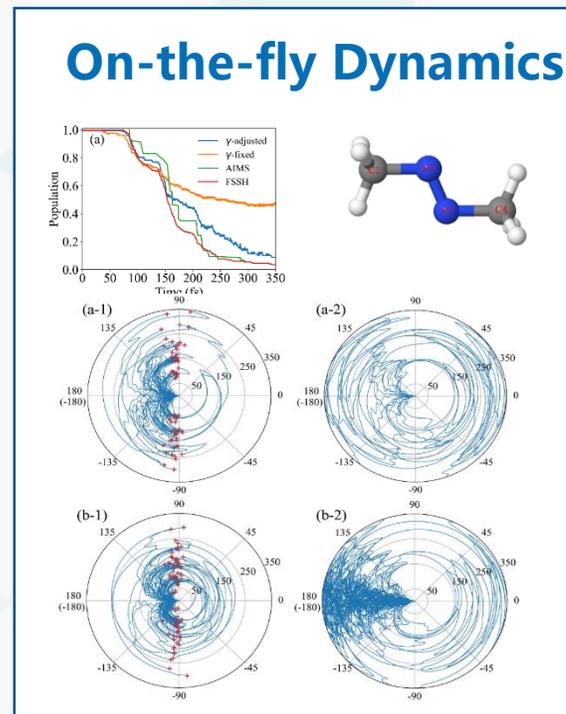
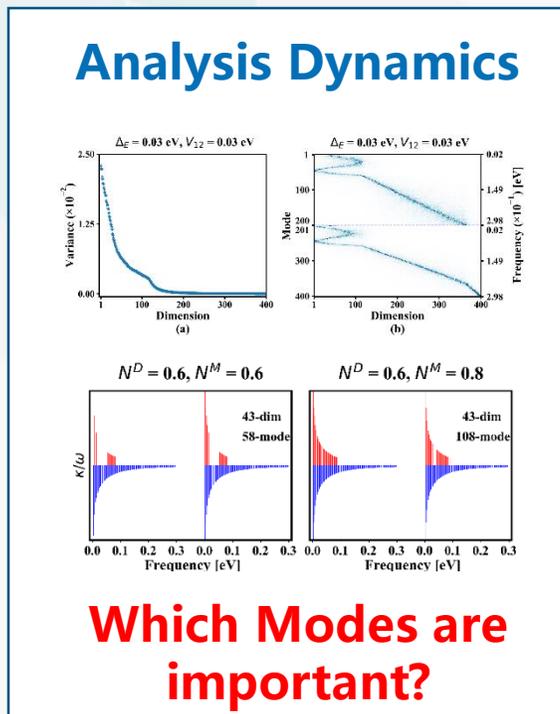
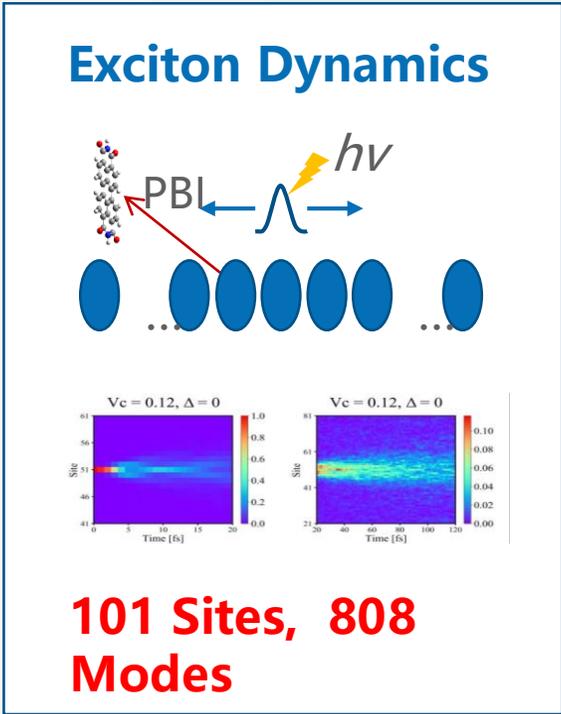
$$\hat{x}_n = (\hat{a}_n^\dagger + \hat{a}_n) / \sqrt{2}$$

$$\hat{p}_n = i(\hat{a}_n^\dagger - \hat{a}_n) / \sqrt{2}$$

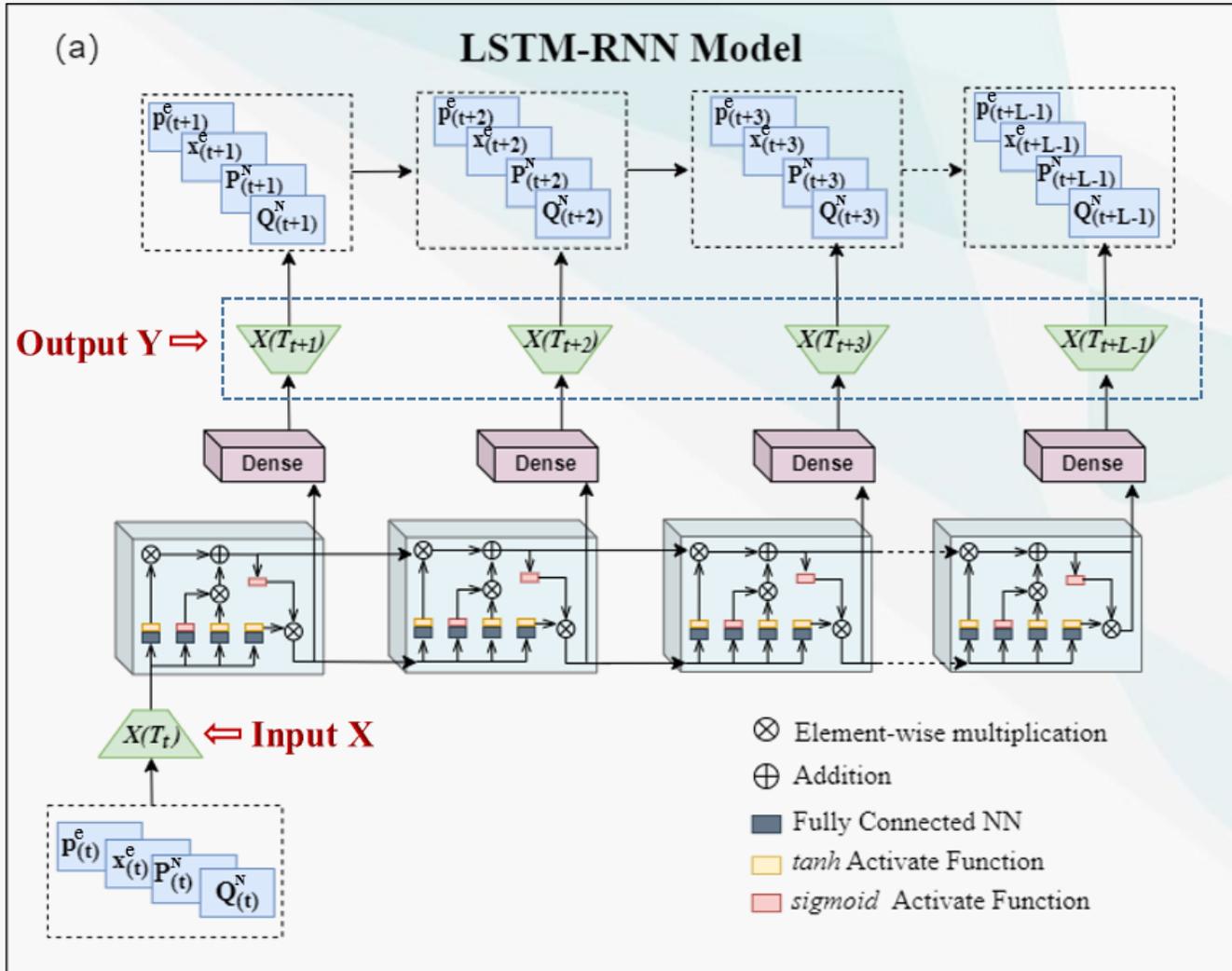
[*J. Chem. Phys.*, 70: 3214-3223 (1979)]

[*Phys. Rev. Lett.*, 78: 578-581 (1997)]

$$\hat{H} = \sum_n \frac{1}{2} (\hat{x}_n^2 + \hat{p}_n^2 - 1) \hat{h}_{nn} + \frac{1}{2} \sum_{n \neq m} (\hat{x}_n \hat{x}_m + \hat{p}_n \hat{p}_m) \hat{h}_{nm}$$

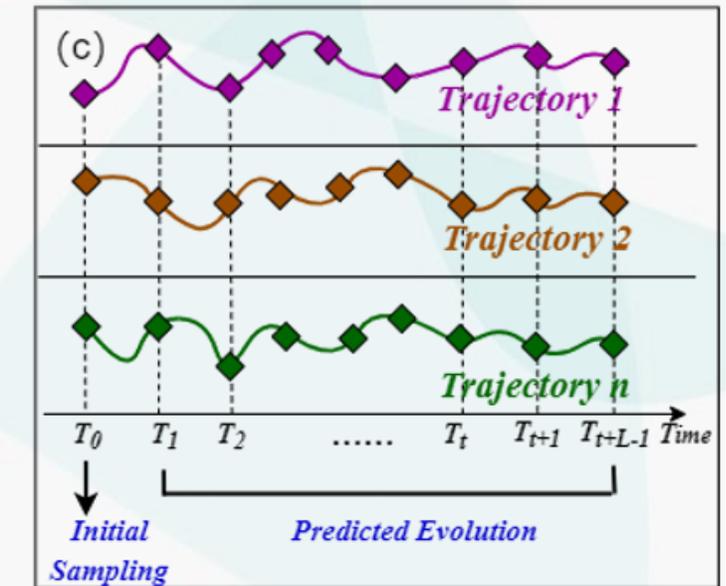
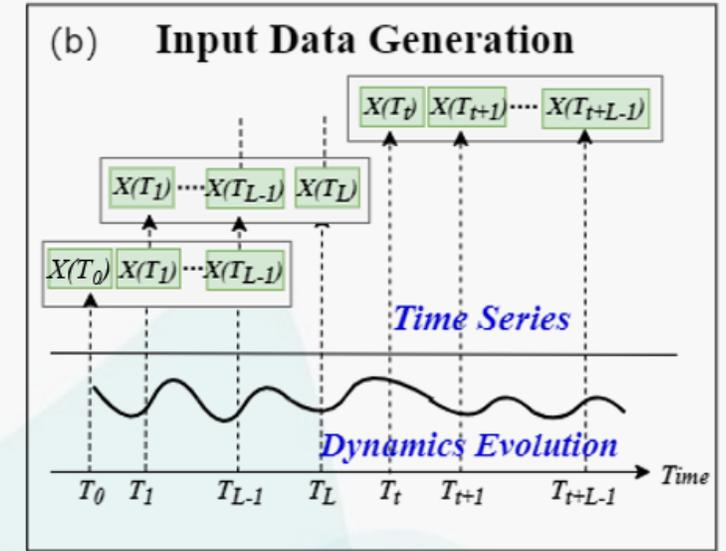


## ➤ Working Process

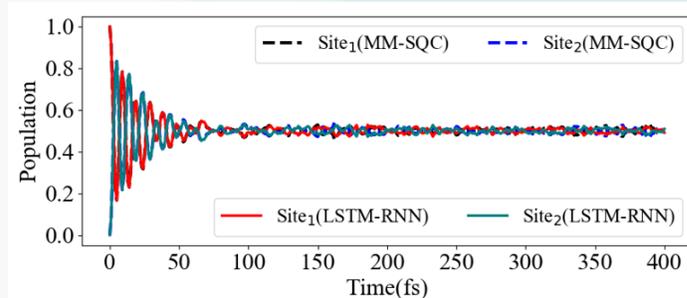


**Training**

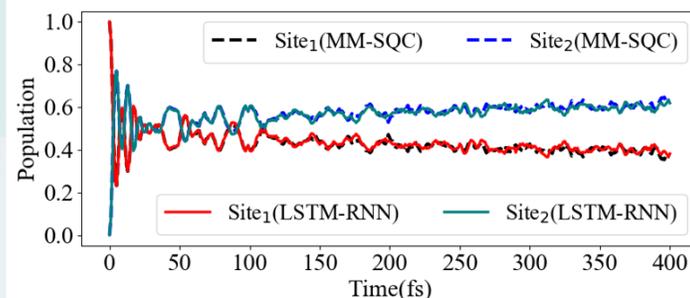
**Prediction**



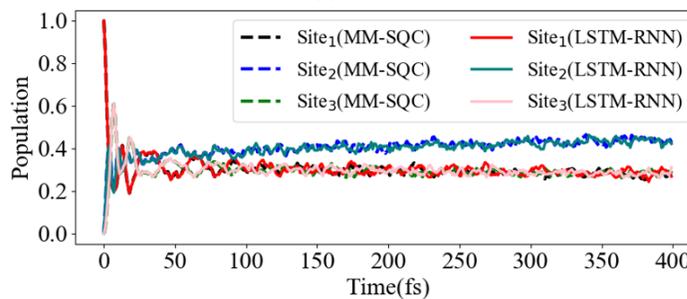
## ➤ Site-exciton models



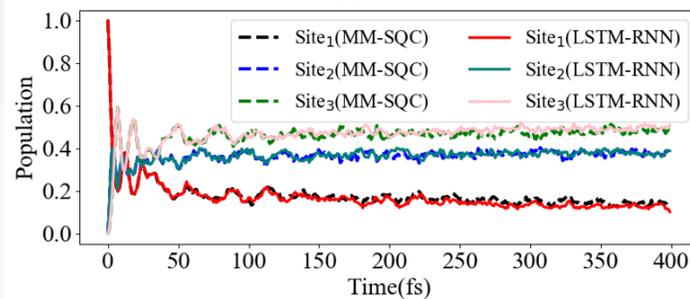
(a) Model I



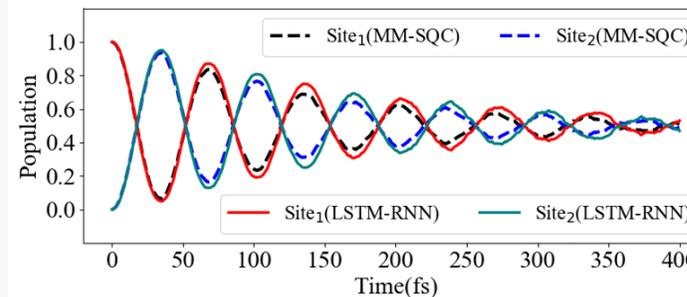
(b) Model II



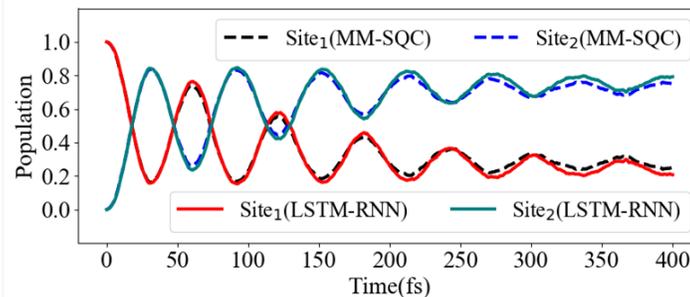
(c) Model III



(d) Model IV

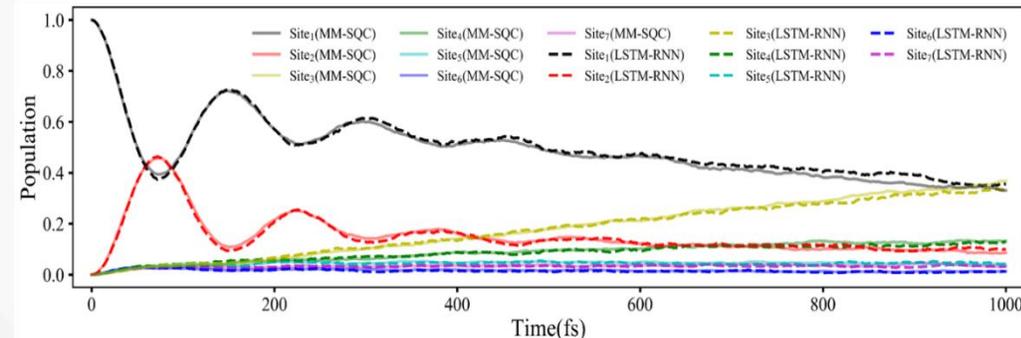


(e) Model V



(f) Model VI

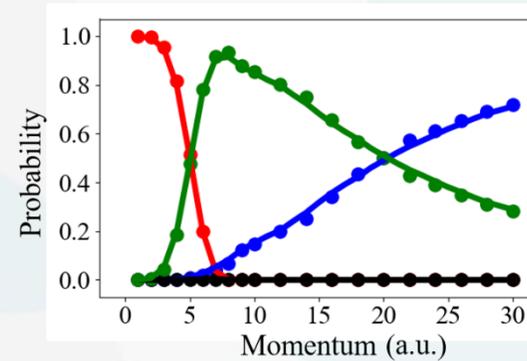
## ➤ FMO System



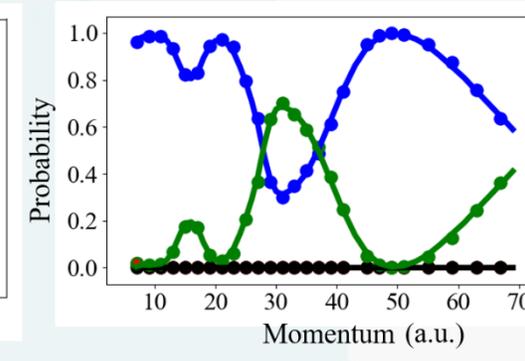
(g) FMO Model

**714 Dimensions**

## ➤ Tully's scattering models



(a) Model VIII (Tully's SAC model)



(b) Model IX (Tully's DAC model)

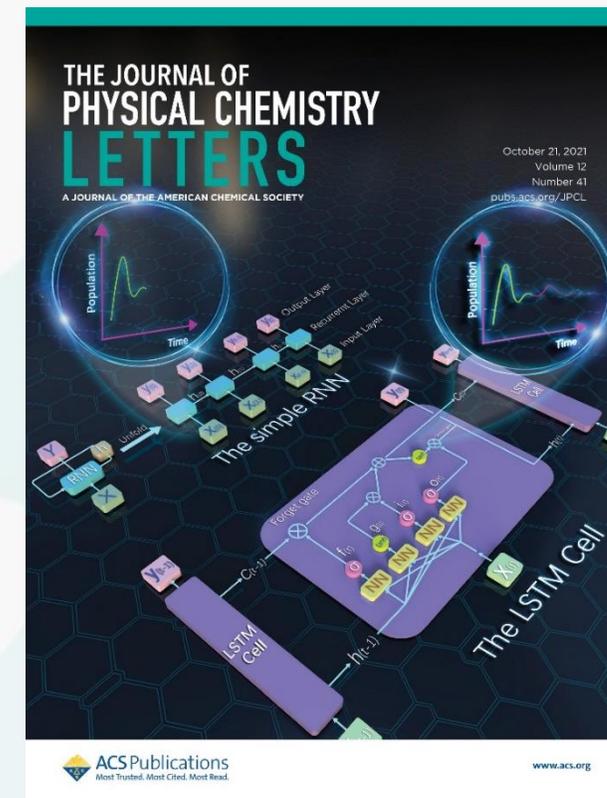
## Time-Series ML Methods

Time-Series Machine Learning Methods  
may be Used to Simulate the  
Nonadiabatic Dynamics

Promising



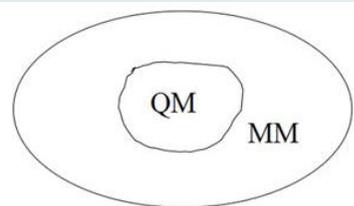
Problems



[Lin, K.; Peng, J.; Gu, F. L.; Lan, Z. *J. Phys. Chem. Lett.*, 12, 10225–10234 (2021)]

[Lin, K.; Peng, J.; Xu, C.; Gu, F. L.; Lan, Z. *J. Chem. Theory Comput.*, 18(10), 5837–5855 (2022)]

[Lin, K.; Peng, J.; Xu, C.; Gu, F. L.; Lan, Z. *J. Phys. Chem. Lett.*, 13, 11678–11688 (2022)]



$$\mathbf{H} = \mathbf{H}_{\text{QM}} + \mathbf{H}_{\text{MM}} + \mathbf{H}_{\text{QM/MM}}$$

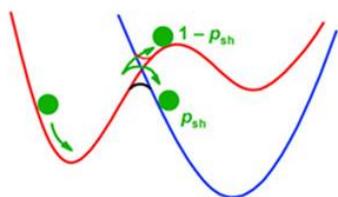
## Excited State Method

CASSCF, ADC(2),  
TDDFT, CIS, TDDFTB

$$\hat{H}_e \chi(\mathbf{r}, \mathbf{R}) = E_e(\mathbf{R}) \chi(\mathbf{r}, \mathbf{R})$$

$$\mathbf{H}\mathbf{C} = \mathbf{S}\mathbf{C}\mathbf{E}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}$$

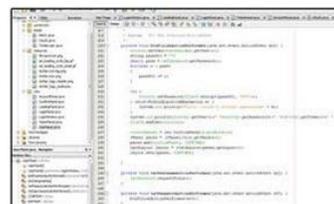


Surface Hopping Dynamics

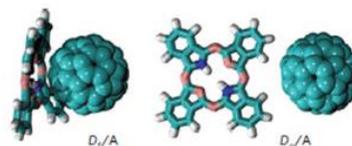
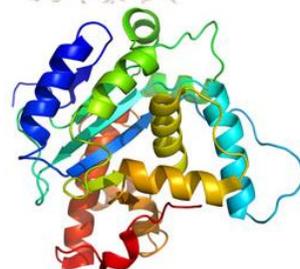
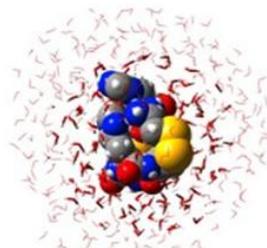
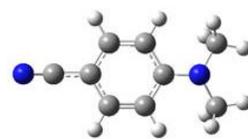
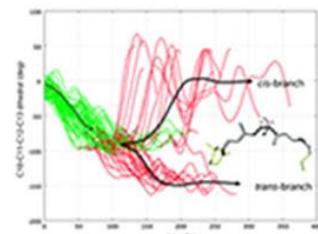
$$i\hbar \frac{dc_j(t)}{dt} = \sum_i c_i(t) [H_{ji} - i\hbar \dot{\mathbf{R}} \cdot \mathbf{d}_{ji}]$$

$$\mathbf{d}_{ji} \equiv \int d\mathbf{r} \phi_j^*(\mathbf{r}, \mathbf{R}) [\nabla_{\mathbf{R}} \phi_i(\mathbf{r}, \mathbf{R})]$$

$$P_{ij} = \frac{2 \int_t^{t+\Delta t} dt \text{Re}(c_i^* c_j \dot{\mathbf{R}} \cdot \mathbf{d}_{ji})}{|c_i(t)|^2}$$



code development



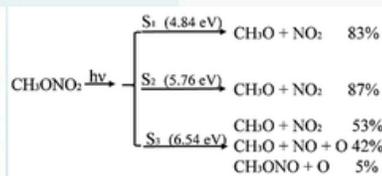
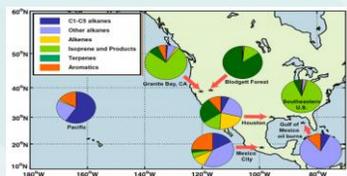
- ◆ On-the-fly Nonadiabatic Dynamics
- ◆ All Degrees of Freedom
- ◆ Black-Box Simulation Tool

## Method and Code Developments:

- ❑ Initial sampling
  - Wigner, Action-angle Sampling
- ❑ Dynamics Module:
  - Surface-hopping dynamics (Tully, Zhu-Nakamura)
  - Quasiclassical Dynamics with Mapping Hamiltonian
- ❑ Electronic Structure Module:
  - TDDFT, CIS, ADC(2), CASSCF, OM2/MRCI, XMS-CASPT2
  - Turbomole, Gaussian, GAMESS, Molpro, MNDO, BAGEL
- ❑ Analytical and Numerical NAC
- ❑ Hybrid QM/MM Methods

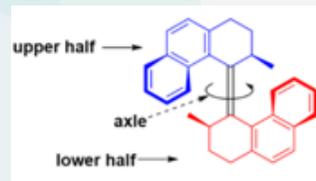
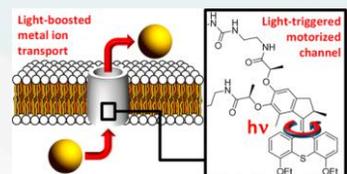
- L. Du, Z. Lan\*, *J. Chem. Theory Comput.*, 2015, 11, 1360;
- D. Hu, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2017, 19, 19168
- D. Hu, Z. Lan\* et al., *J. Chem. Theory Comput.*, 2021, 17, 3279

## Environmental Photochemistry



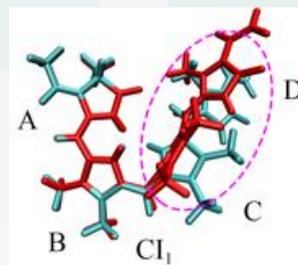
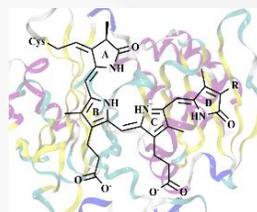
- J. Zhang, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2021, 23, 25597
- S. Lin, F.G. Gu\*, Z. Lan\* et al., *J. Chem. Phys.*, 2021, 155, 214105
- K. Lin, F.G. Gu\*, Z. Lan\* et al., *Chemosphere.*, 2021, 281, 130831
- X. Kang, Z. Lan\* et al., *Chin. J. Chem. Phys.*, 2023

## Photoisomerization



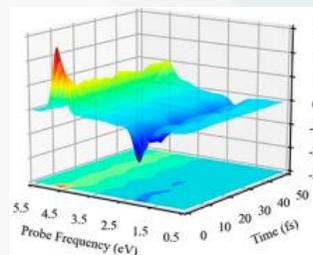
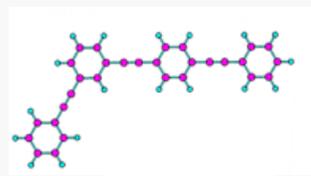
- J. Ma, C. Jiang\*, Z. Lan\* et al., *Int. J. Mol. Sci.*, 2022, 23, 9694
- J. Ma, C. Jiang\*, Z. Lan\* et al., *Int. J. Mol. Sci.*, 2022, 23, 3908
- C. Xu, Z. Lan\* et al., *J. Phys. Chem. Lett.*, 2022, 13, 661
- D. Hu, Z. Lan\* et al., *J. Chem. Theory Comput.*, 2021, 17, 3267

## Photobiology



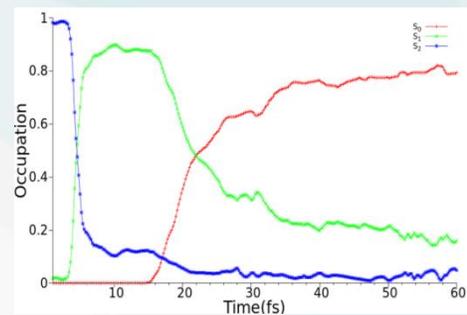
- Y. Fang, F.G. Gu\*, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2022, 24, 26190
- H. Huang, C. Xu\*, Z. Lan\* et al., *Chin. Chem. Lett.*, 2022, 107850
- Y. Zhu, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2022, 24, 24362
- D. Hu, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2017, 19, 19168

## Photovoltaics

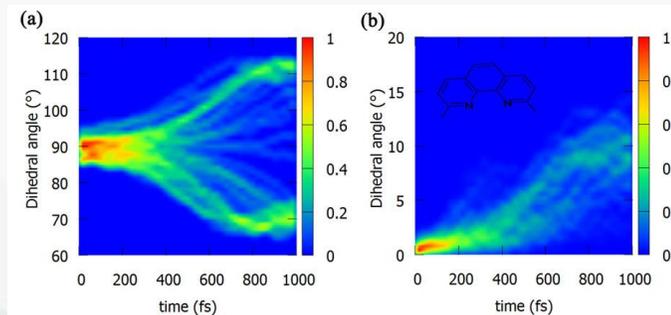


- D. Hu, Z. Lan\* et al., *J. Phys. Chem. Lett.*, 2021, 12, 9710
- J. Zheng, Y. Xie\*, Z. Lan\* et al., *Phys. Chem. Chem. Phys.*, 2020, 22, 18192
- Y. Xie, H. Ren\*, Z. Lan\* et al., *J. Mater. Chem. A*, 2019, 7, 27484
- S. Jiang, Y. Xie\*, Z. Lan\* et al., *Chem. Phys.*, 2018, 515, 603
- Y. Xie, Z. Lan\* et al., *J. Chem. Phys.*, 2018, 149, 174105

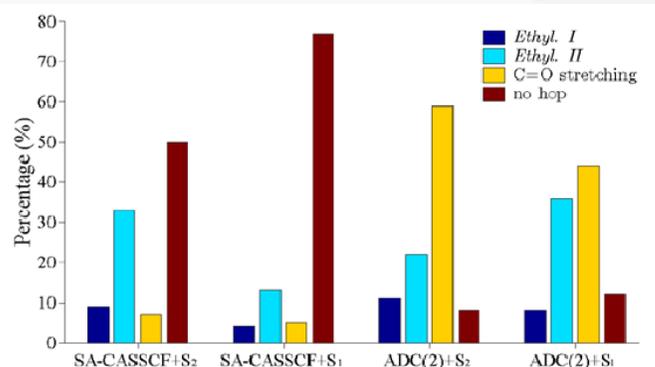
## Population dynamics Lifetime



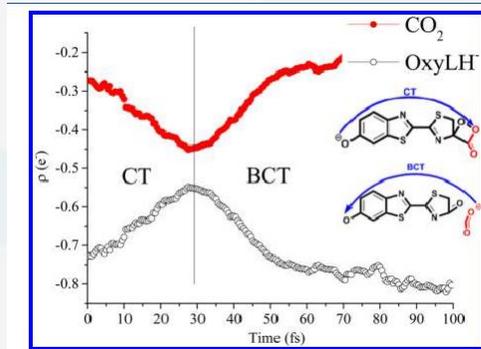
## Geometry evolution



## Reaction channels

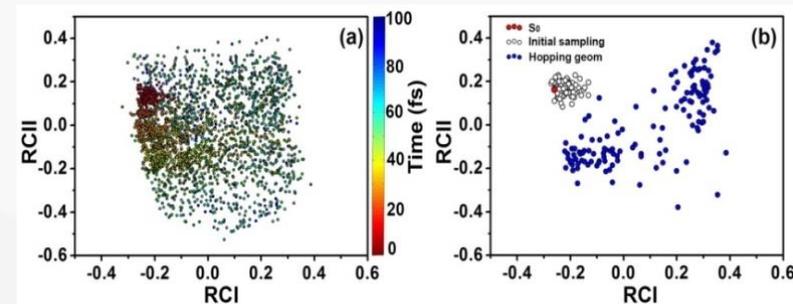
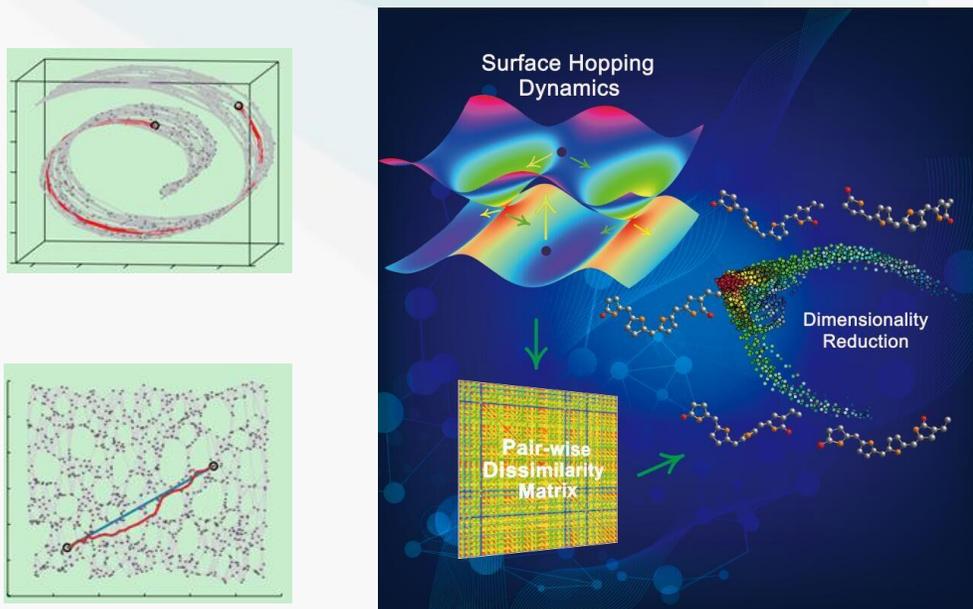


## Physical quantities

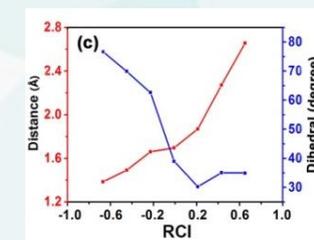
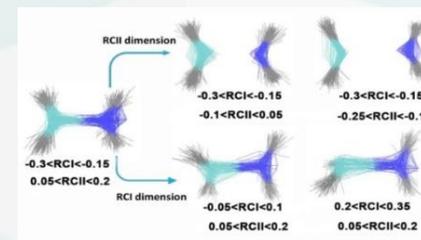


# Analysis of trajectory evolution I: geometrical evolution 25

- Dimensionality reduction approaches to analyze the surface-hopping dynamics simulation results
- Extract the major molecular motion



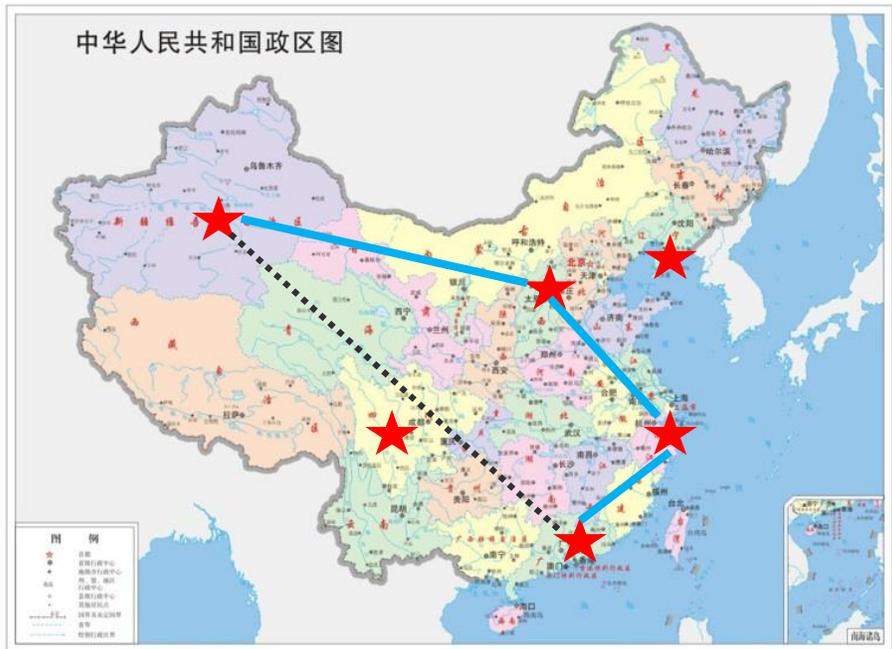
Dimensionality reduction  
 $\{R_N\} \rightarrow \{r\}$



- A large number of trajectories
- Polyatomic molecules
- Many degrees of freedom

- **Multidimensional scaling**
- **Isometric feature mapping**

## How to draw a map from the inter-city distances?

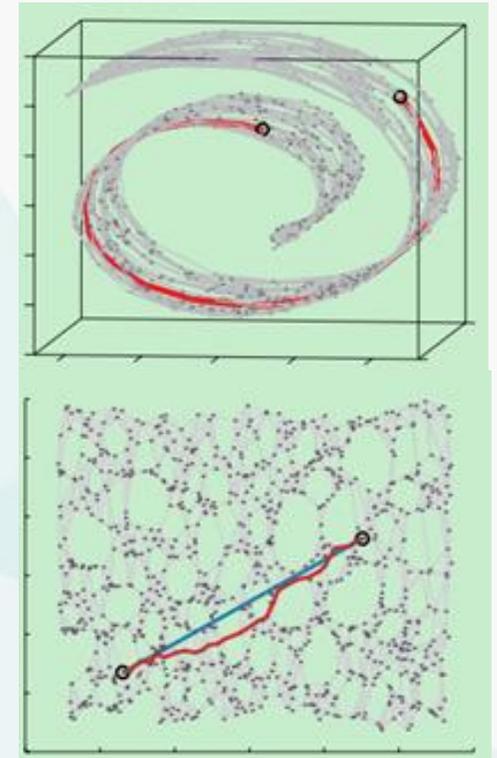


MDS (Multidimensional Scaling)

|   | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A |   |   |   |   |   |   |
| B |   |   |   |   |   |   |
| C |   |   |   |   |   |   |
| D |   |   |   |   |   |   |
| E |   |   |   |   |   |   |
| F |   |   |   |   |   |   |

Inter-city Distance Matrix (I, J)

ISOMAP (Isometric Mapping)

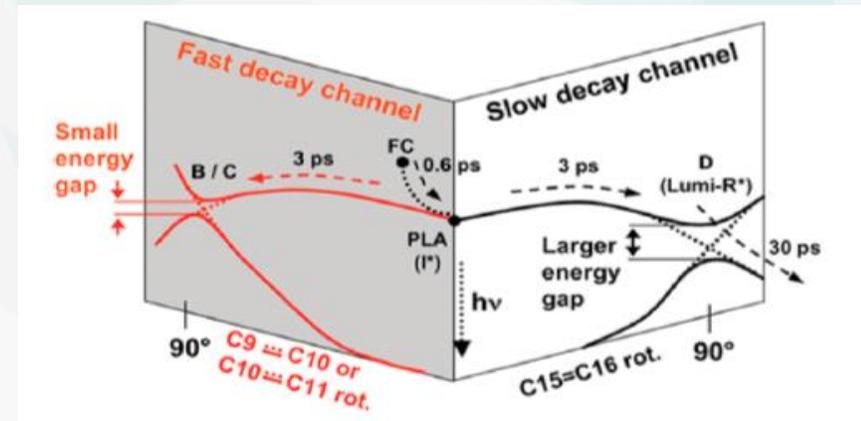
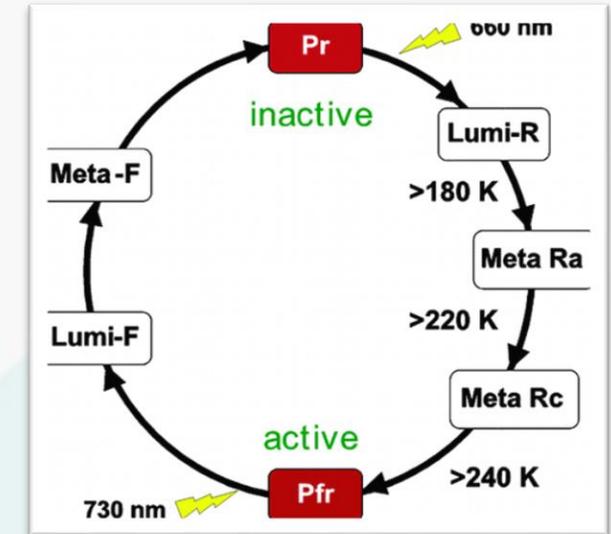
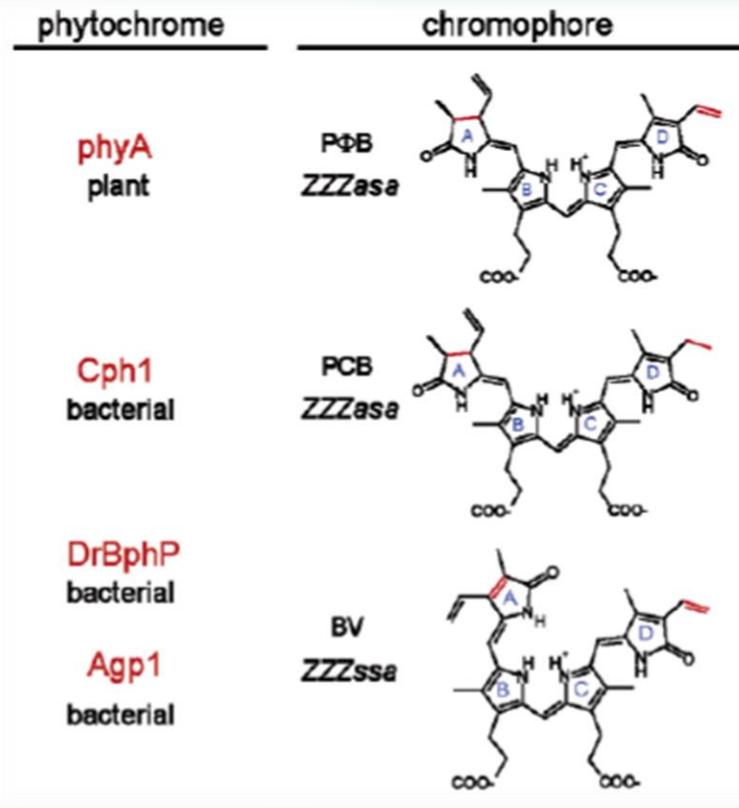


- [1] Hardle, W.; Simar, L. *Springer*. Berlin, **2007**.
- [2] Borg, I.; Groenen, P. J. F. *Springer Science & Business Media: America*, **2005**.
- [3] De Silva, V.; Tenenbaum, J. B.; *Technical report*. Stanford University, **2004**.
- [4] Balasubramanian, M.; Schwartz, E. L. *Science* **2002**, 295.
- [5] Tenenbaum, J. B.; de Silva, V.; Langford, J. C. *Science* **2000**, 290, 2319-2323.

- Euclidean distance  $\rightarrow$  Geodesic distance
- K-points or Epsilon-ball
- Dijkstra or Floyd-Warshall algorithm

# Example: ZaZsZa isomer of PΦB model

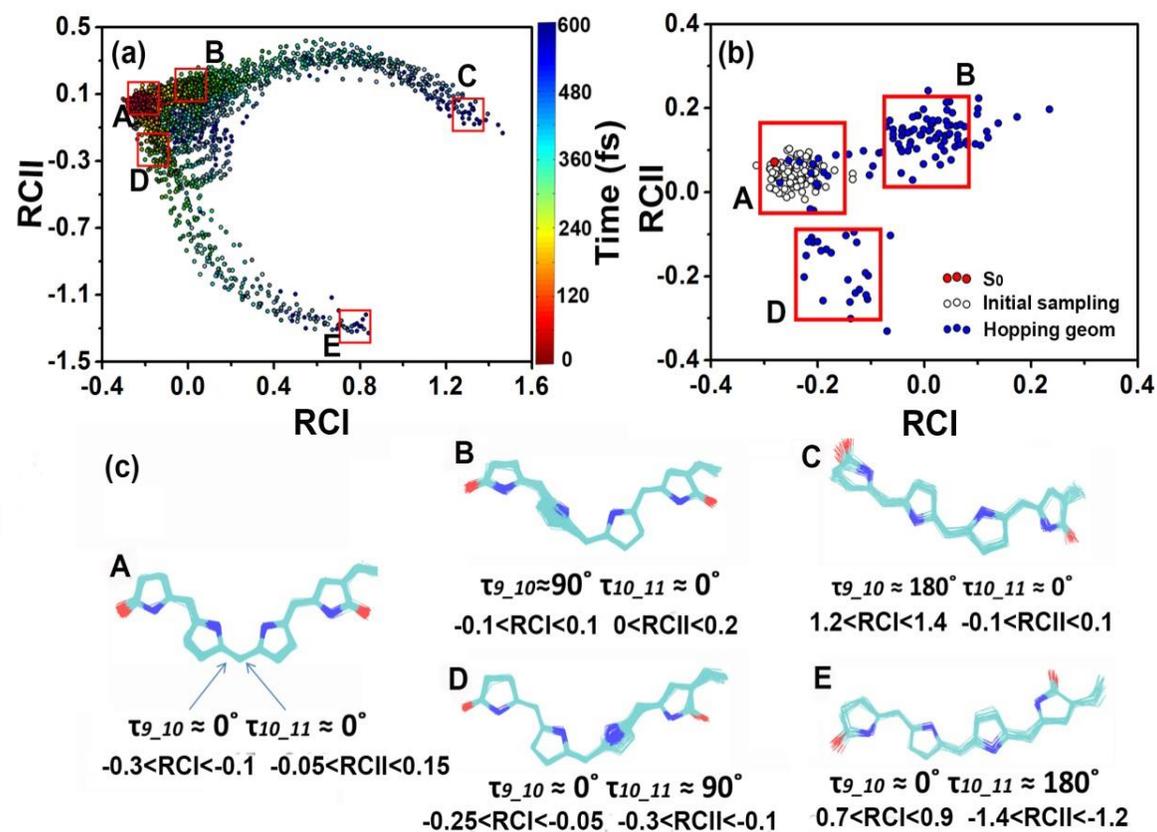
## The PΦB model



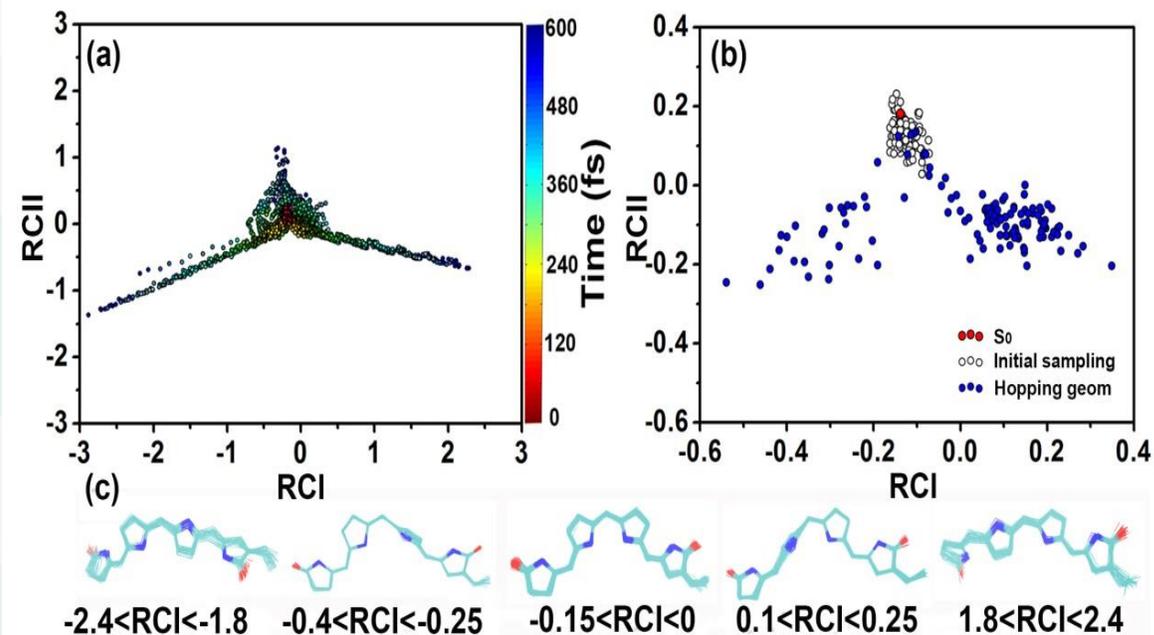
1. Maria A. M., Daniel H. M., Peter H. *Acc. Chem. Res.*, **2007**, 40 (4), pp 258–266
2. Samer G., Hoi L. L., Igor S., Olivucci M. *Chem. Rev.* **2017** DOI: 10.1021/acs.chemrev.7b00177

# Example: ZaZsZa isomer of PΦB model

## MDS

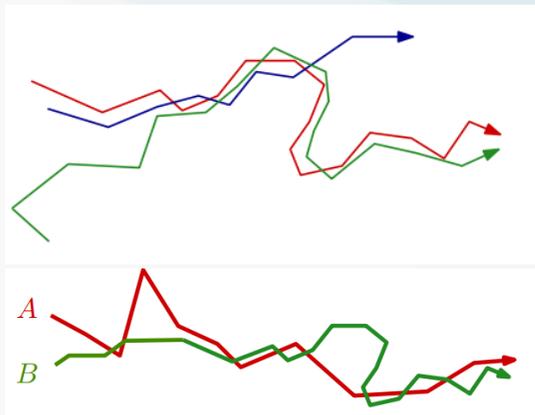


## ISOMAP

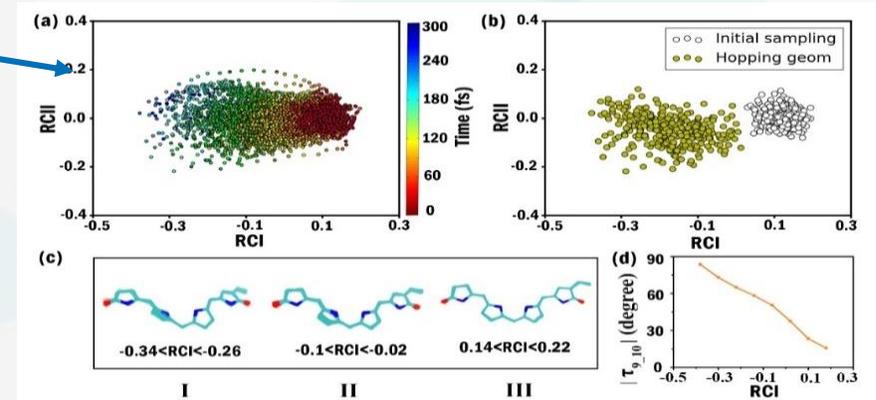
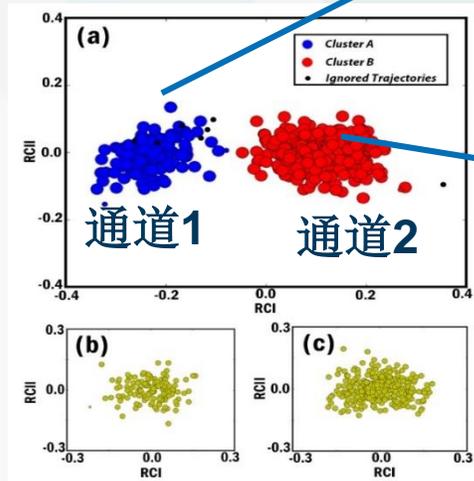
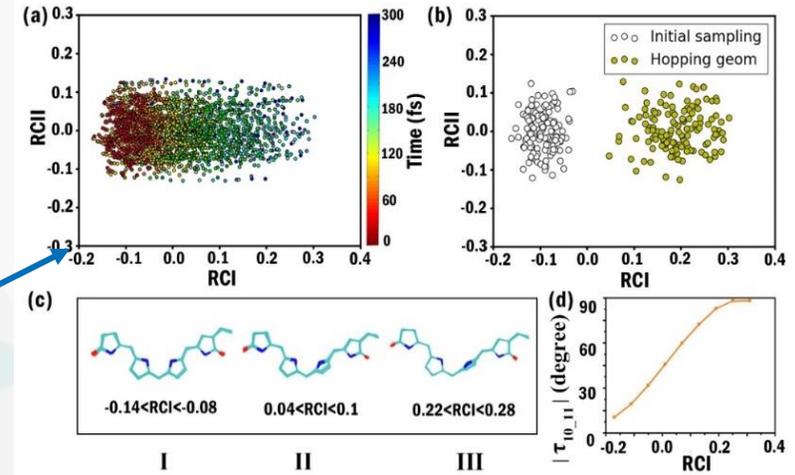
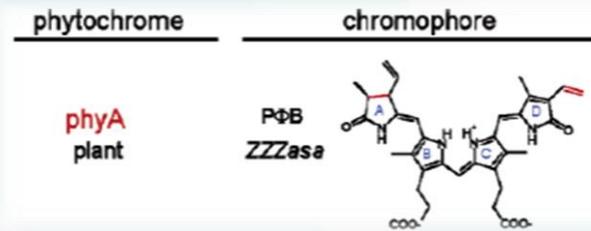
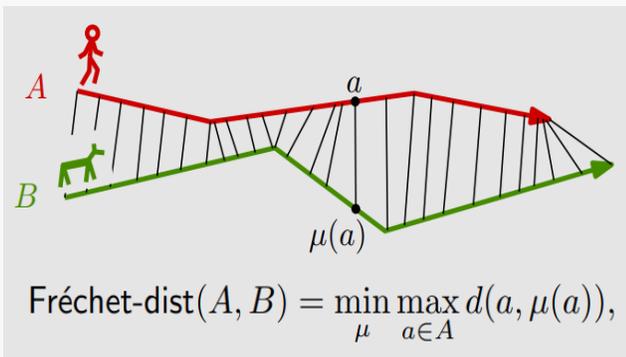


# Analysis of trajectory evolution II: trajectory similarity

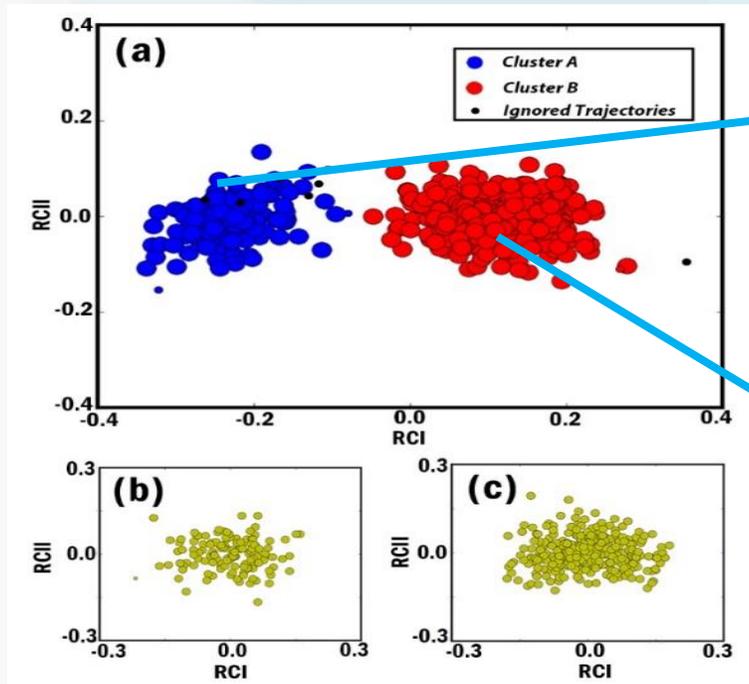
- An “automatic” approach to analyze the trajectory similarity and the configuration similarity in the on-the-fly trajectory surface hopping dynamics.



## Fréchet Distance



## Clustering Analysis of Trajectory Similarity before $S_1$ - $S_0$ Hops

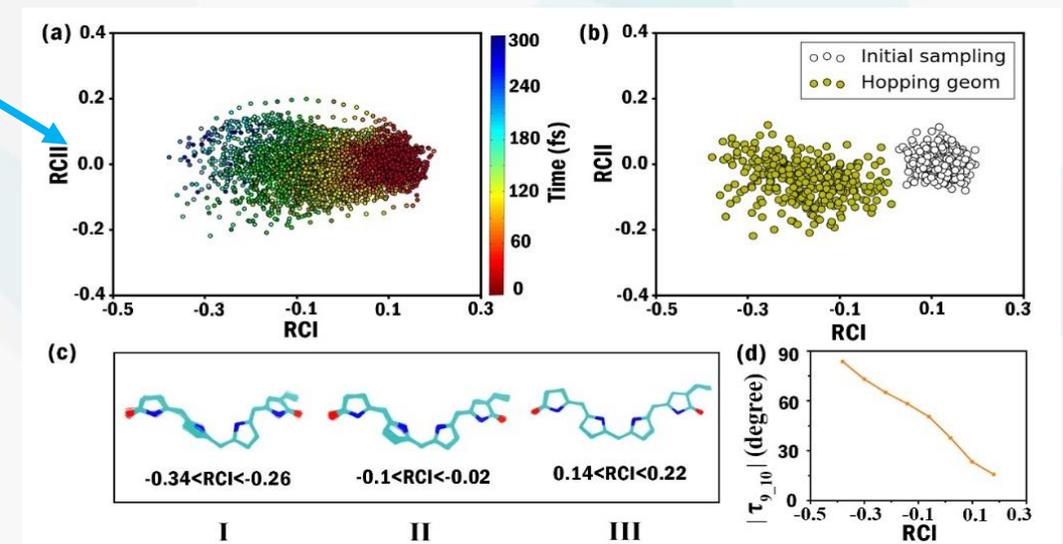
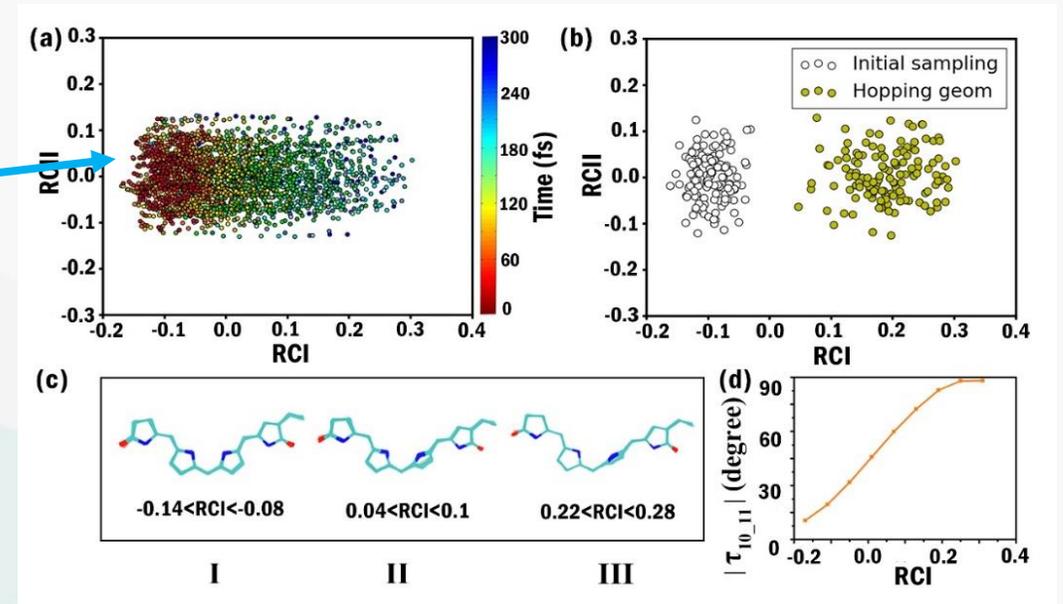


Cluster A

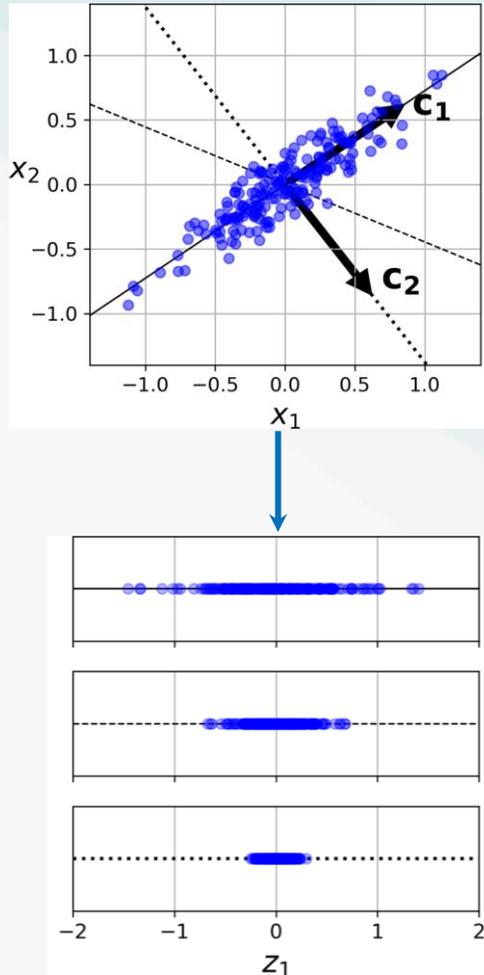
Cluster B

- I. Get dynamics results
- II. Select geometries
- III. Calculate trajectory distance
- IV. Clustering
- V. Dimensionality reduction analysis

- X. Li, D. Hu, Y. Xie, Y. Lan\*, JCP, 149, 244104, 2018



## Principal Component Analysis



### ➤ Geometry Data Collections

The MM-SQC Dynamics

$$H_{MM}(x, p, Q, P) = \sum_k \left[ \frac{1}{2} (x_k^2 + p_k^2) - \gamma \right] H_{kk}(Q, P) + \frac{1}{2} \sum_{k \neq l} (x_k x_l + p_k p_l) H_{kl}(Q, P)$$

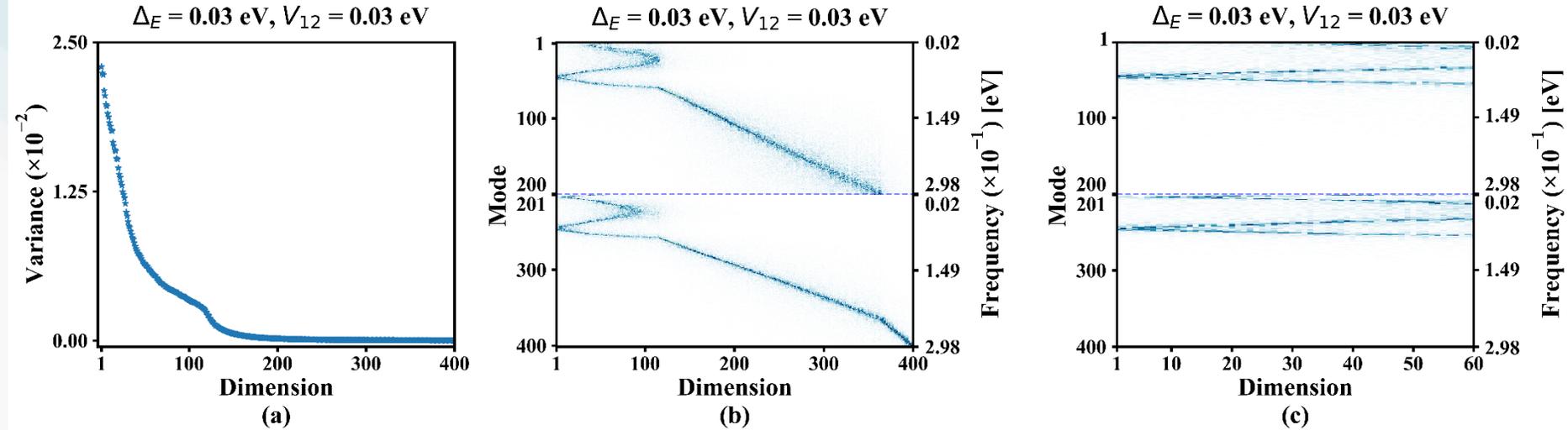
### ➤ Descriptor Construction

- The Action Variable of the Bath Mode:  $N_j = \frac{1}{2} Q_j^2 + \frac{1}{2} P_j^2$
- The Coordinate of the Bath Mode:  $Q_j$

### ➤ Dimensionality Reduction

$$\mathbf{M}_{CO} = (\mathbf{X} - \langle \mathbf{X} \rangle)^T (\mathbf{X} - \langle \mathbf{X} \rangle) = \mathbf{U}^T \mathbf{E} \mathbf{U}$$

## The Action Variable of the Bath Mode



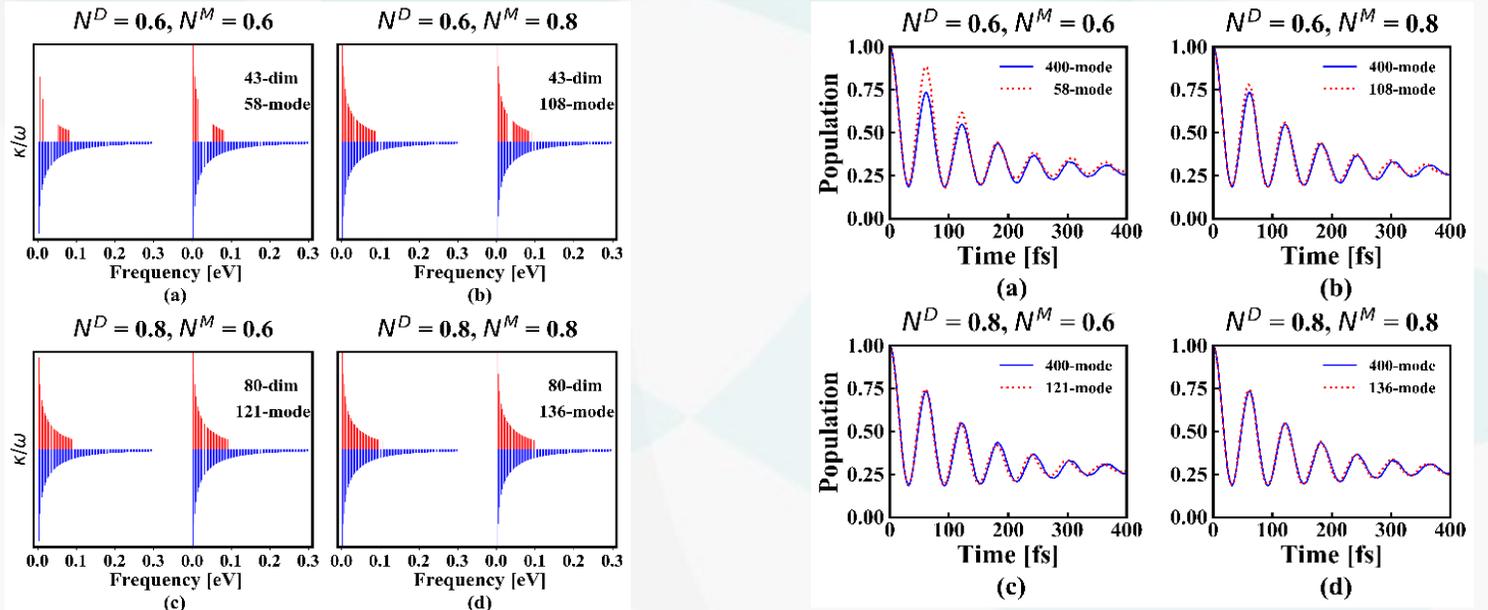
## Reduced Model Construction

### ◆ Select Dimension

$$N^D \sim \sum_i E_i$$

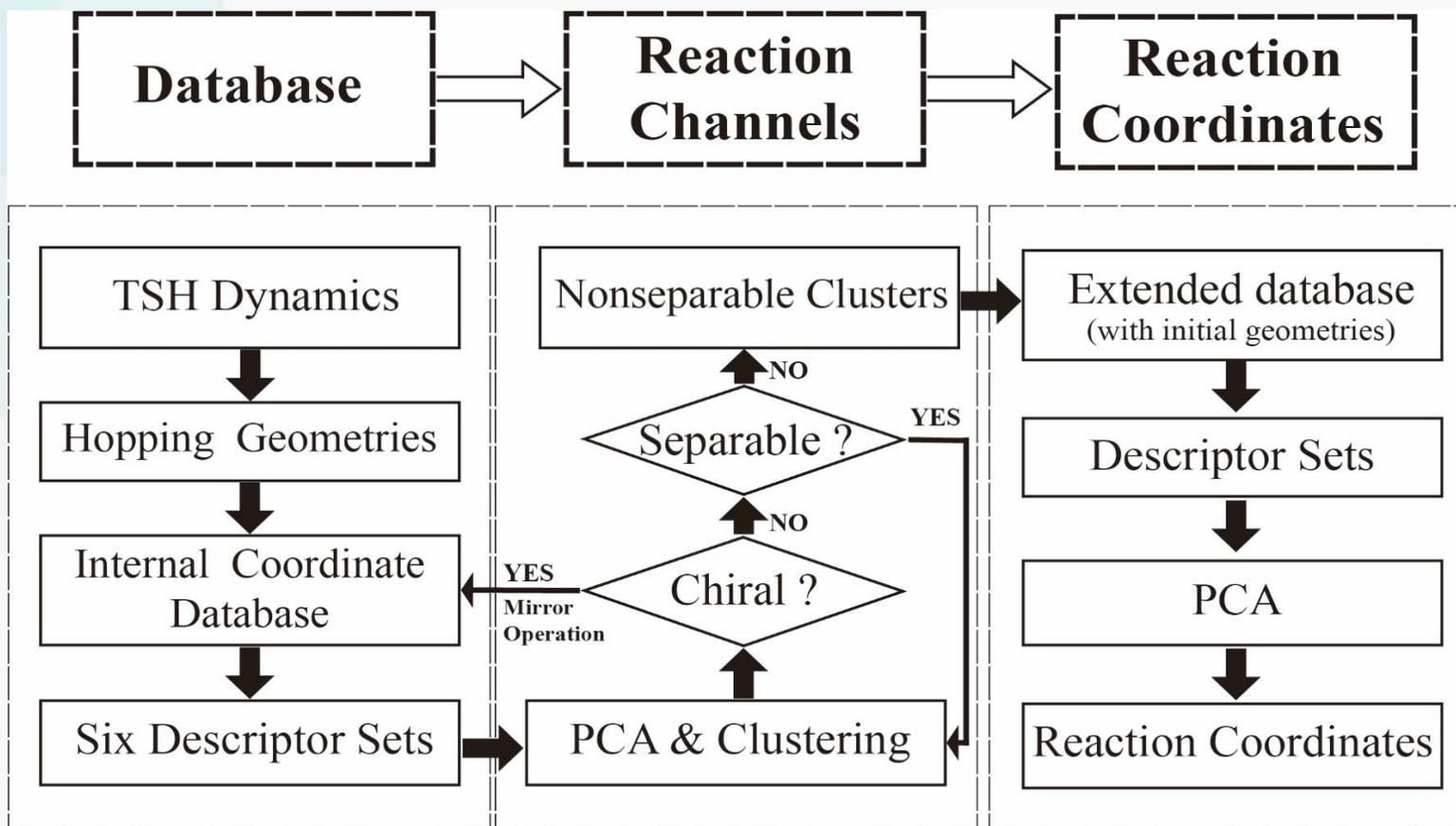
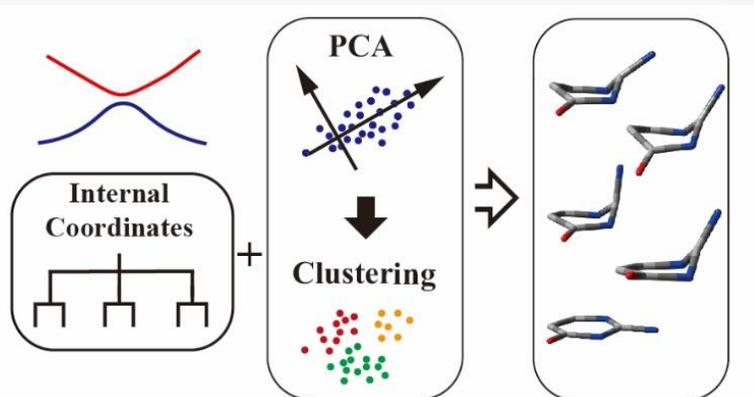
### ◆ Select Modes

$$N_i^M \sim \sum_j |U_{ij}|^2$$



An **hierarchical** protocol based on the PCA and clustering methods for the **automatic** analysis of the **ring deformation** in the nonadiabatic dynamics

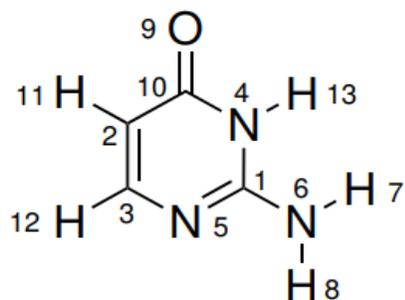
- Dimensionality reduction : PCA
- Clustering : DBSCAN & Agglomerative clustering



## Division scheme and analysis process

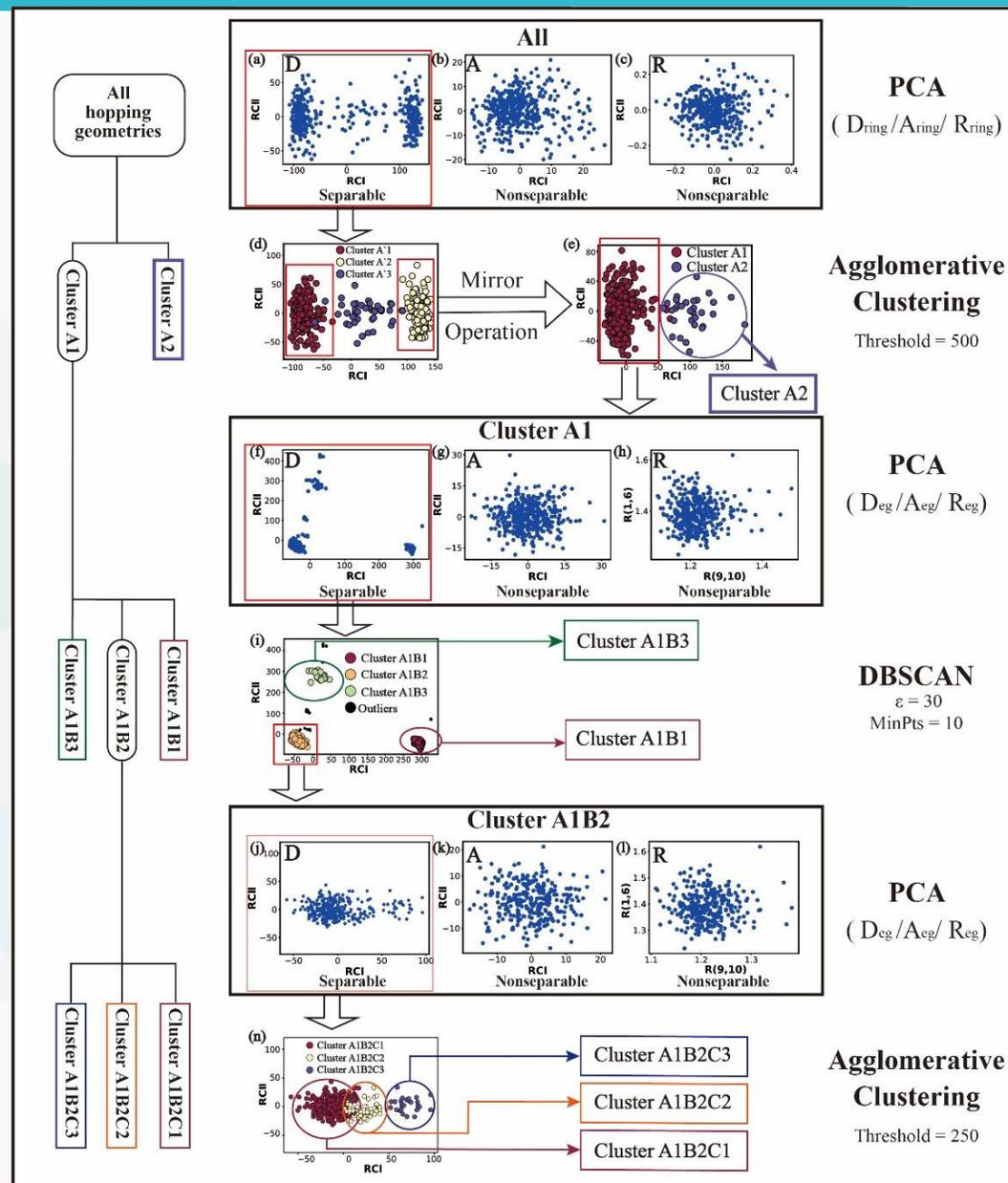
Molecular Structure and Atomic Labels

Keto-  
isocytosine



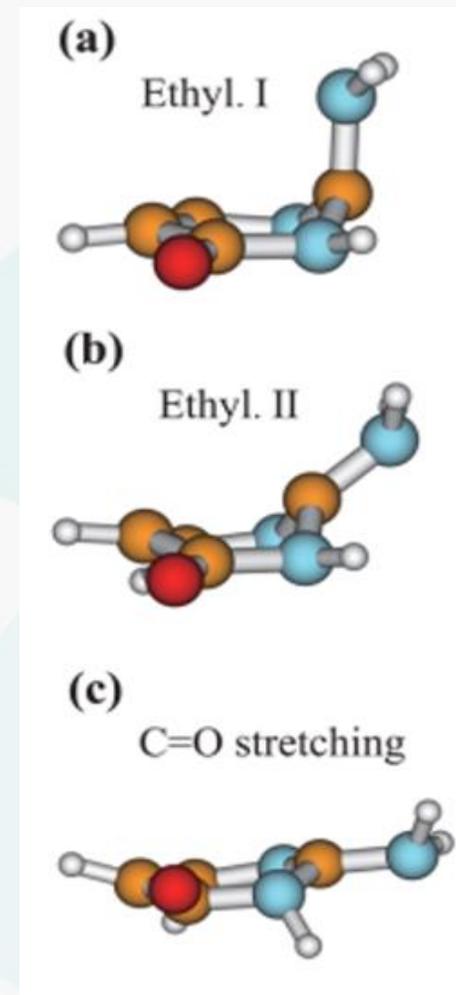
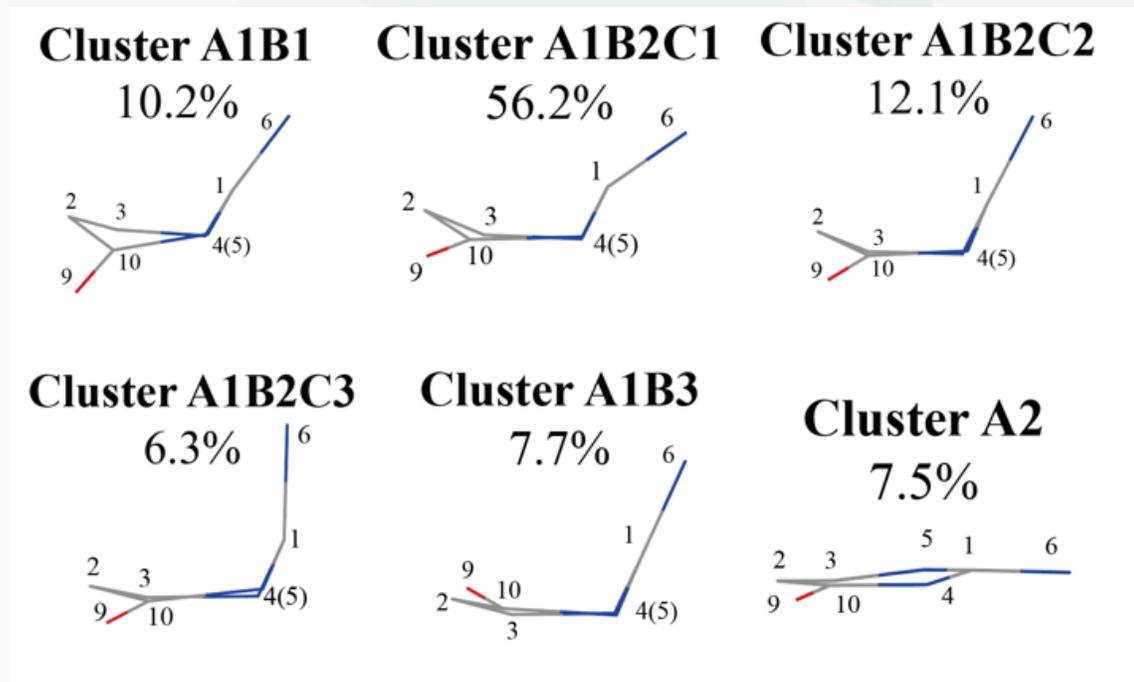
Descriptor Sets

|                   |            |             |             |             |
|-------------------|------------|-------------|-------------|-------------|
| $D_{\text{ring}}$ | D(4,1,5,3) | D(5,1,4,10) | D(10,2,3,5) | D(3,2,10,4) |
|                   | D(4,1,5,3) | D(5,1,4,10) |             |             |
| $A_{\text{ring}}$ | A(4,1,5)   | A(3,2,10)   | A(2,3,5)    | A(1,4,10)   |
|                   | A(1,5,3)   | A(2,10,4)   |             |             |
| $R_{\text{ring}}$ | R(1,4)     | R(1,5)      | R(2,3)      | R(4,10)     |
|                   | R(3,5)     | R(2,10)     |             |             |
| $D_{\text{eg}}$   | D(6,1,5,3) | D(6,1,4,10) | D(1,4,10,9) | D(3,2,10,9) |
|                   |            |             |             |             |
| $A_{\text{eg}}$   | A(4,1,6)   | A(2,10,9)   | A(5,1,6)    | A(4,10,9)   |
|                   |            |             |             |             |
| $R_{\text{eg}}$   | R(1,6)     | R(9,10)     |             |             |
|                   |            |             |             |             |



## Identify reaction coordinates

- Key active coordinates (**major and minor**)
- Other related information (ratios etc.)
- Further physical insights



## Un-supervised

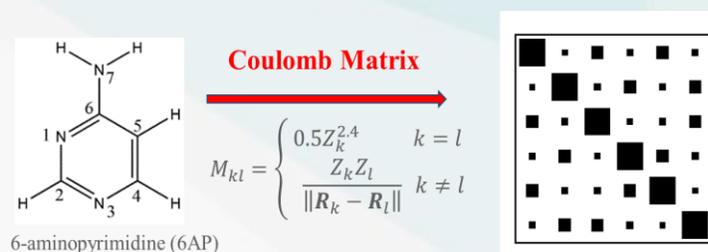
Machine Learning Methods  
may Bring Considerable Impact on  
Nonadiabatic Dynamics Simulation

Promising

?

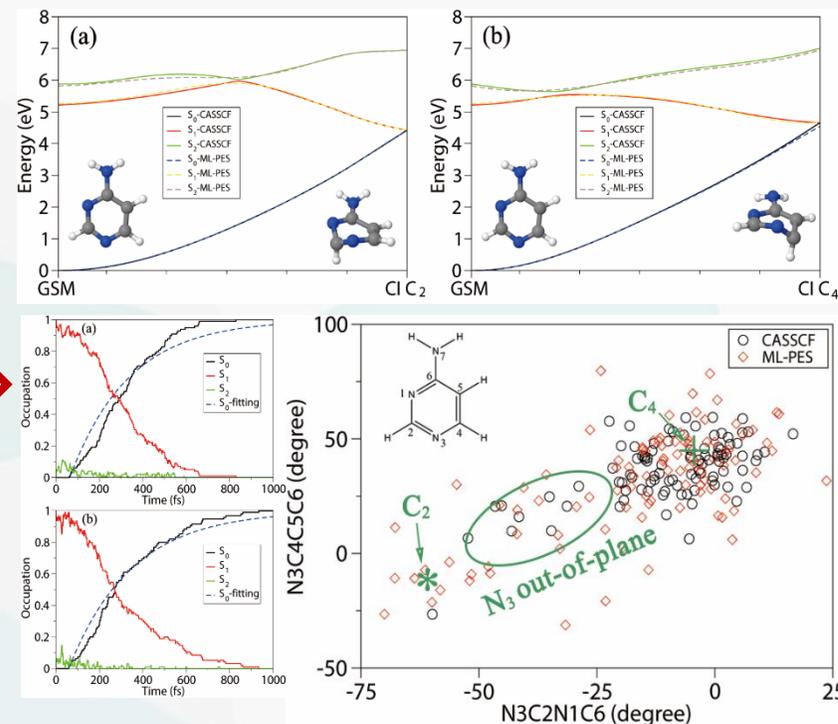
Problems

- The kernel ridge regression is used to build the excited-state PESs
- Nonadiabatic dynamics based on ML-PESs



$$f(\mathbf{m}_i) = \sum_{j=1}^{N_t} c_j K(\mathbf{m}_i, \mathbf{m}_j)$$

$$K(\mathbf{m}_i, \mathbf{m}_j) = \exp\left(-\frac{\|\mathbf{m}_i - \mathbf{m}_j\|^2}{2\sigma^2}\right)$$



- Achieve the efficient massive dynamics simulations with a large number of trajectories.

## Hamiltonian

$$\hat{H}(t) = \hat{H}_M + \hat{H}_F(t)$$

$$\hat{H}_F(t) = -\hat{\mu} \cdot \mathbf{E}(t)$$

## Third-order polarization

$$P^{(3)}(t) = (i)^3 \int_0^\infty dt_3 \int_0^\infty dt_2 \int_0^\infty dt_1 \mathbf{E}(t-t_3) \mathbf{E}(t-t_3-t_2) \mathbf{E}(t-t_3-t_2-t_1) S(t_3, t_2, t_1)$$

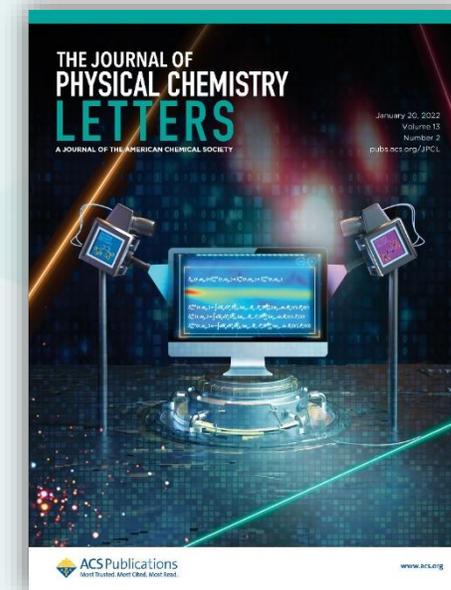
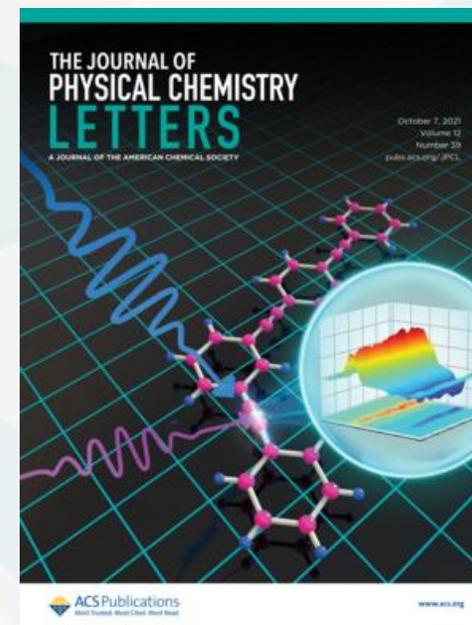
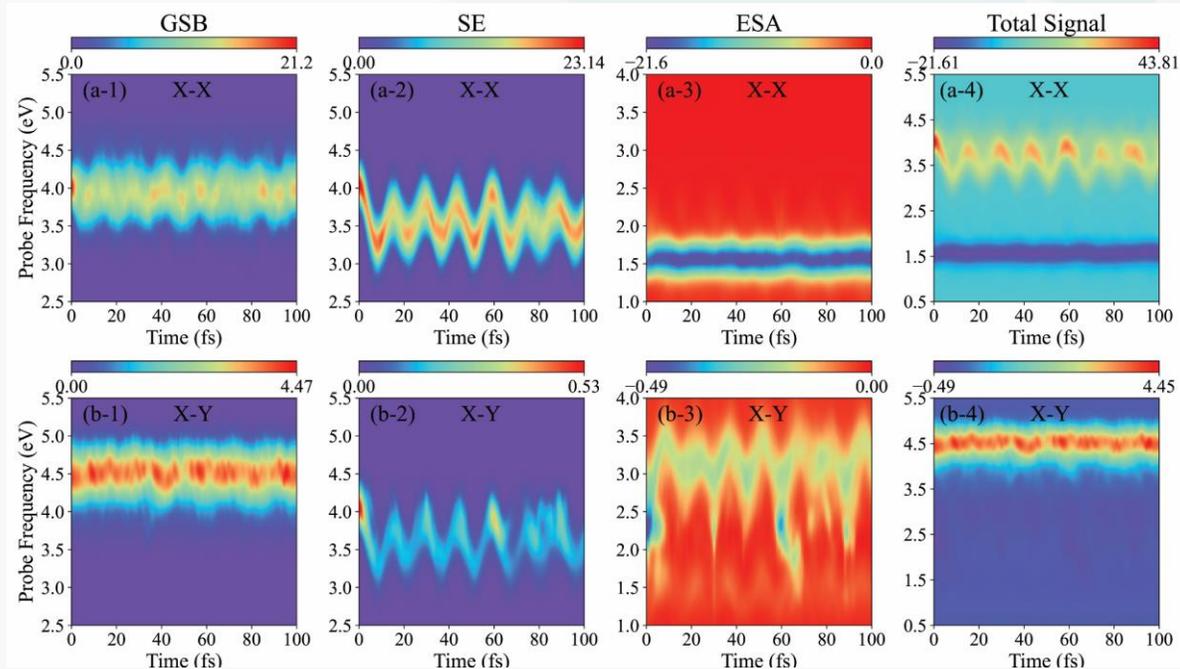
$$S(t_3, t_2, t_1) = \text{Tr}\{\hat{\mu}'(t_1+t_2+t_3)[\hat{\mu}'(t_1+t_2), [\hat{\mu}'(t_1), [\hat{\mu}'(0), \hat{\rho}(-\infty)]]]\}$$

$$\hat{\mu}'(t) = e^{i\hat{H}_M(t-t_0)} \hat{\mu} e^{-i\hat{H}_M(t-t_0)}$$

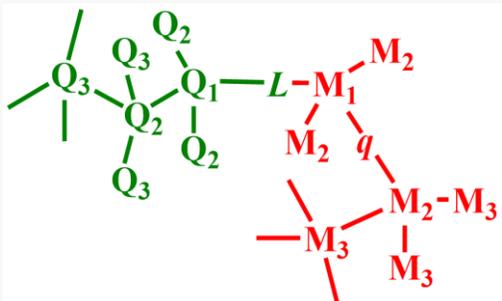
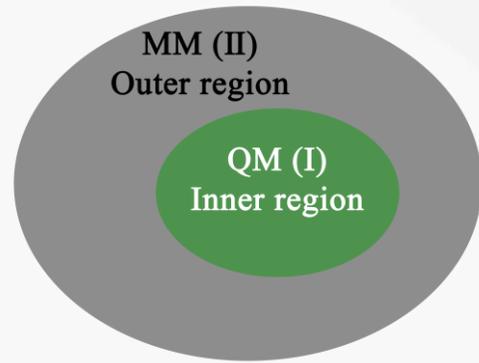
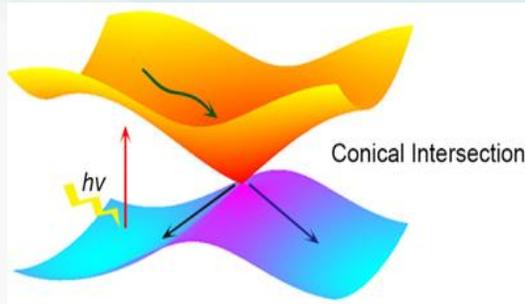
## Pump-probe signal

$$I_{\text{int}}(\tau, \omega_{pr}) = \omega_{pr} \text{Im} \left\{ \int_{-\infty}^{\infty} dt E_{pr}(t) e^{i\omega_{pr}t} P_{k_{pr}}^{(3)}(\tau, t) \right\}$$

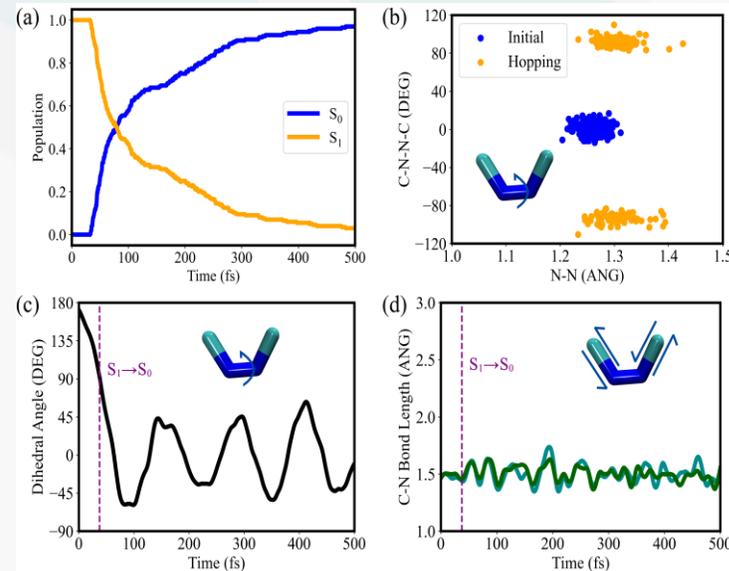
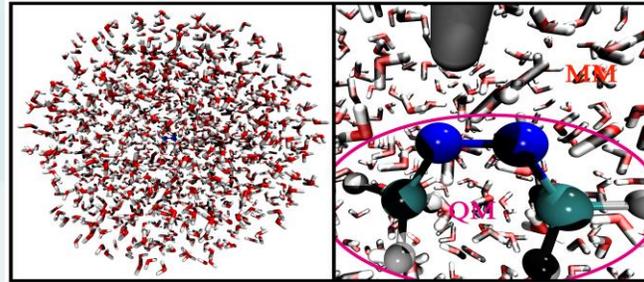
$$I_{\text{dis}}(\tau, \omega) = \omega_{pr} \text{Im} \left\{ \varepsilon_{pr}(\omega) P_{k_{pr}}^{(3)}(\tau, \omega) \right\}$$



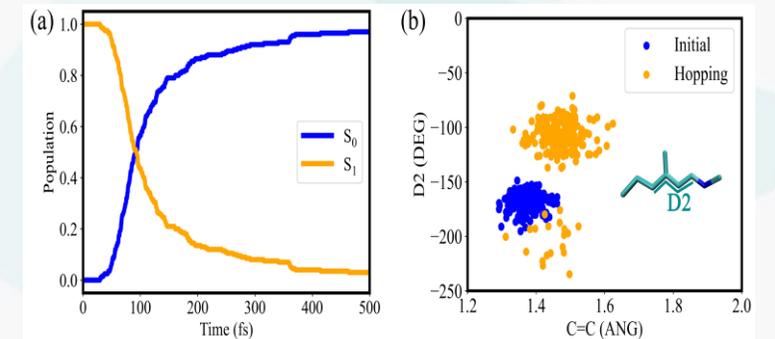
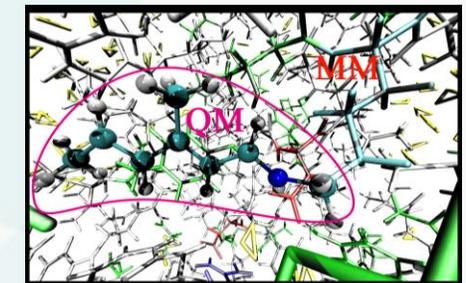
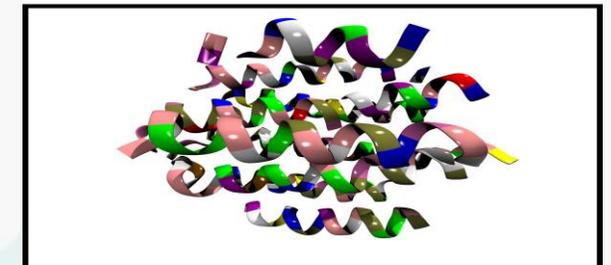
## QM/MM Surface Hopping



## Nonadiabatic Dynamics in Solutions



## Nonadiabatic Dynamics in Proteins



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in China**



**City view of  
Guangzhou**



**South China  
Normal University**



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**New Postdoc and Research Assistant Positions are open !!!**



华南师范大学  
SOUTH CHINA NORMAL UNIVERSITY

# Thank you !

Zhenggang Lan  
2024.02



