



Model Construction and TD-DMRG Simulation for Exciton Dynamics in Molecular Aggregated Systems

Haibo Ma

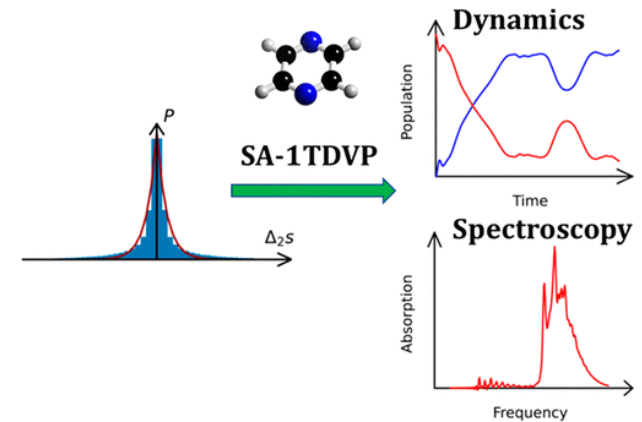
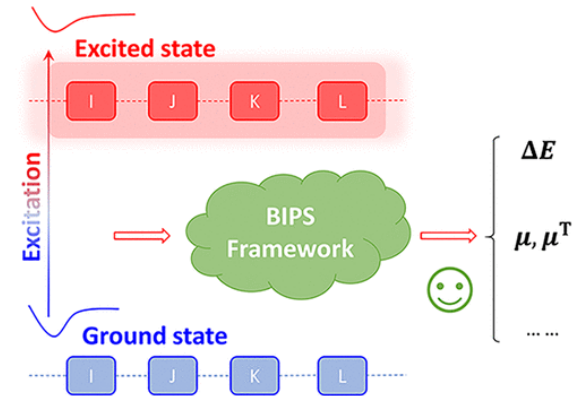
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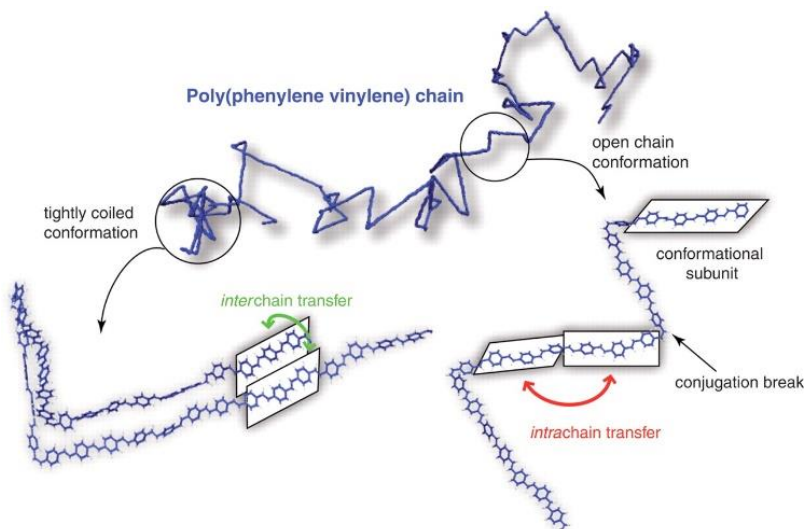
haibo@nju.edu.cn

Outline

- **Background: Exciton dynamics**
- **Model construction**
 - Automatic construction of excitonic basis
 - Evaluation of exciton-phonon couplings
- **Quantum dynamics simulation**
 - Time-dependent density matrix renormalization group (TD-DMRG)
 - Stochastic adaptive single-site TD-DMRG

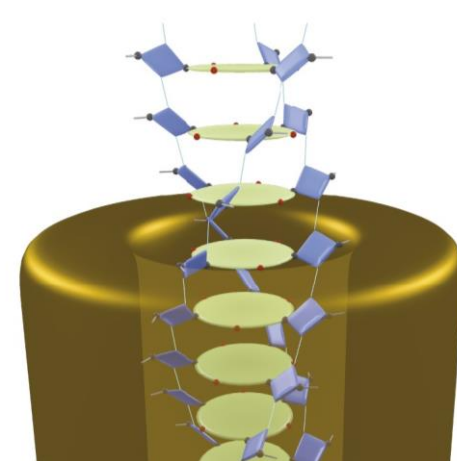
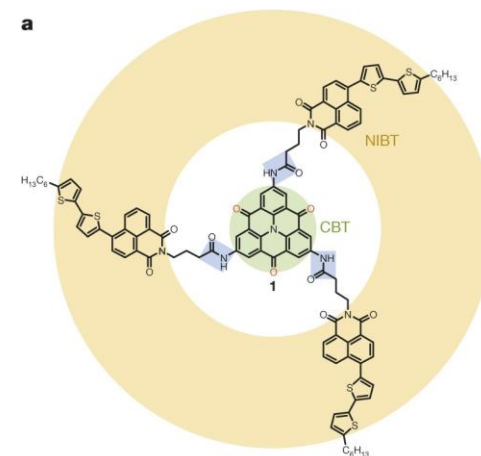


Excited States in Large Photoactive Systems



Intrachain Energy Transfer in a Conjugated Polymer

E. Collini, G. D. Scholes, *Science* **2009**, 323, 369



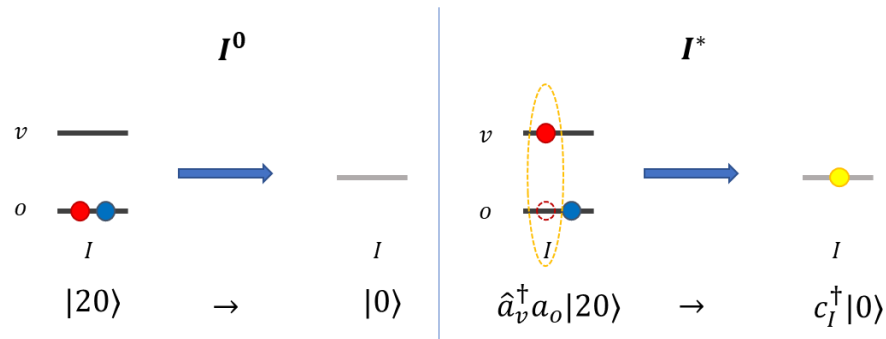
Long-range energy transport in single supramolecular nanofibres

A. T. Haedler, et al., *Nature* **2015**, 523, 196

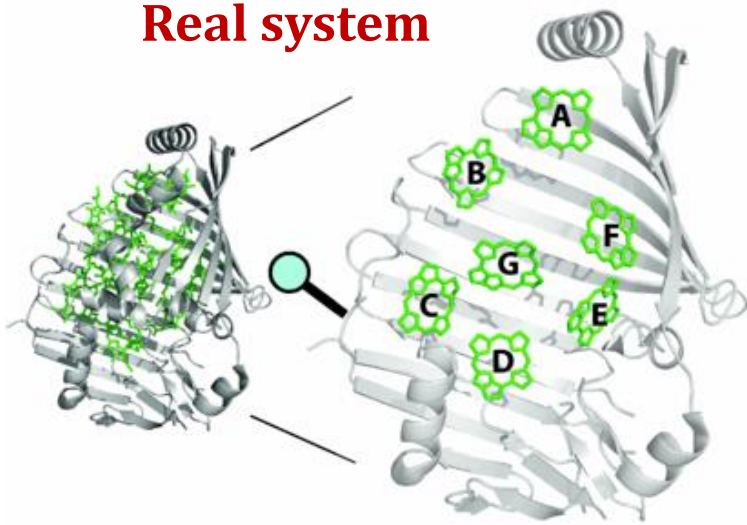
Both the electronic structure and quantum dynamics of large photoactive systems can hardly be simulated by standard quantum chemistry methods.

Excitonic Model Hamiltonians

Exciton: a bound electron-hole pair \rightarrow a quasi-particle



Real system



Fenna-Matthews-Olson (FMO)
bacteriochlorophyll complex

Simplification



Excitonic model

$$\hat{H}_{\text{ex}} = \sum_{i=1}^7 \varepsilon_i \hat{c}_i^\dagger \hat{c}_i + \sum_{i \neq j} V_{ij} \hat{c}_i^\dagger \hat{c}_j$$

(7×7) matrix

Excitonic Model Hamiltonians

To further account for the **exciton-vibration couplings**.

$$\hat{H} = \hat{H}_{\text{ex}} + \hat{H}_{\text{vib}} + \hat{H}_{\text{ex-vib}}$$

$$\hat{H}_{\text{ex}} = \sum_{i=1}^{N_{\text{ex}}} \epsilon_i \hat{c}_i^\dagger \hat{c}_i + \sum_{i \neq j} V_{ij} \hat{c}_i^\dagger \hat{c}_j$$

$$\hat{H}_{\text{vib}} = \sum_{I=1}^{N_{\text{vib}}} \omega_I \hat{b}_I^\dagger \hat{b}_I$$

$$\hat{H}_{\text{ex-vib}} = \sum_{i,j,I} g_{ij}^I \hat{c}_i^\dagger \hat{c}_j (\hat{b}_I^\dagger + \hat{b}_I) + \sum_{i,j,I,J} g_{ij}^{IJ} \hat{c}_i^\dagger \hat{c}_j (\hat{b}_I^\dagger + \hat{b}_I) (\hat{b}_J^\dagger + \hat{b}_J)$$

$$N_{\text{ex}} \sim 10^{0-1}$$
$$N_{\text{vib}} \sim 3 \times N_{\text{atom}} \gg 10^{0-1}$$

Challenges:

1. **Model construction:** How to build the basis and evaluate the parameters?
2. **Quantum dynamics:** How to simulate strongly correlated systems with large N_{vib} ?

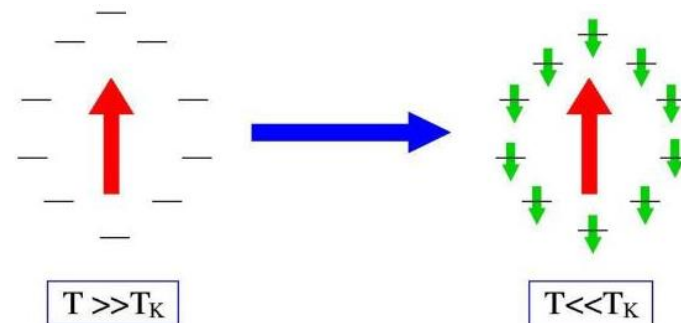
Basis Selection

Numerical renormalization group (NRG)

K. G. Wilson, *Phys. Rev. B* **1971**, 4, 3174. (1982 Nobel prize in physics)

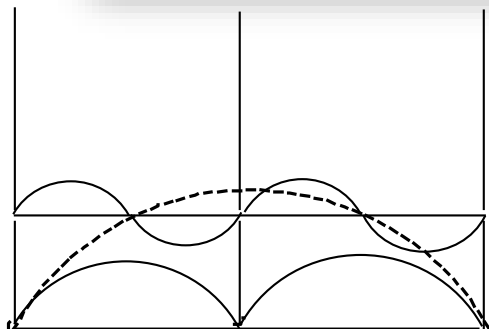


Keep only M energetically lowest states

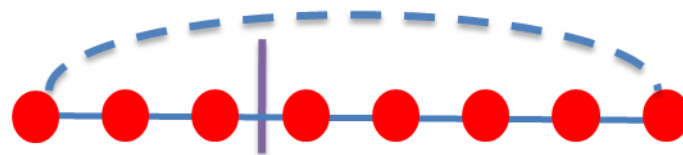
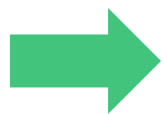


Kondo impurity model

NRG works well in weakly correlated systems, but fails in moderately and strongly correlated systems.



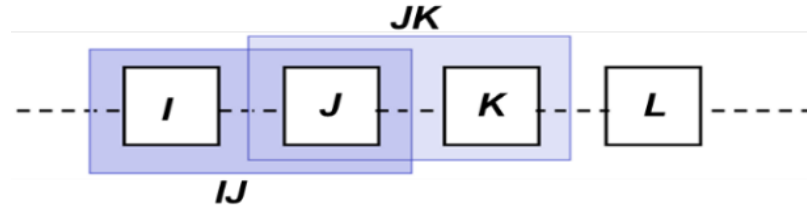
NRG basis describes the inter-subsystem boundaries poorly.



Density Matrix Renormalization Group (DMRG)

S. White, *Phys. Rev. Lett.* **1992**, 69, 2863.

Renormalized Excitonic Model (REM)



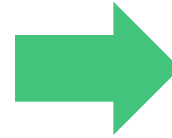
NRG

REM

M. Al Hajj, J.-P. Malrieu, N. Guihéry, *Phys. Rev. B* **2005**, 72, 224412



$$\hat{H} = \sum_I \hat{H}_I + \sum_{IJ} \hat{H}_{I-J} + \dots$$



$$\hat{H}^{\text{REM}} = \sum_I \hat{H}_I^{\text{eff}} + \sum_{IJ} \hat{H}_{I-J}^{\text{eff}} + \dots$$

Improve the description of inter-fragment interaction
through **Bloch's effective Hamiltonian theory**

adiabatic states $|\text{orange}\rangle$ $|\text{yellow}\rangle$

$$\begin{pmatrix} \varepsilon_{IJ}^* & 0 \\ 0 & \varepsilon_{IJ}^{**} \end{pmatrix}$$

Target space S

diabatic states $|\text{orange} \otimes \text{blue}\rangle$ $|\text{blue} \otimes \text{orange}\rangle$

$$\begin{pmatrix} H_I^{\text{eff}} & H_{IJ}^{\text{eff}} \\ H_{IJ}^{\text{eff}} & H_J^{\text{eff}} \end{pmatrix}$$

Model space S_0

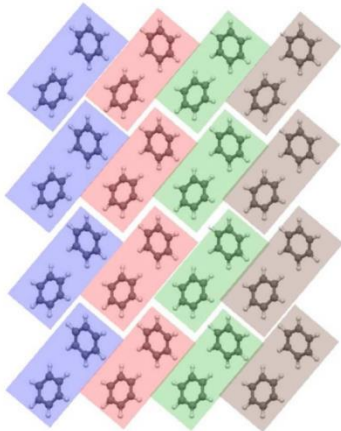
7

H. Zhang, J. P. Malrieu, **H. Ma**, J. Ma, *J. Computat. Chem.* **2012**, 33, 34.

Y. Ma, Y. Liu, **H. Ma**, *J. Chem. Phys.* **2012**, 136, 224412.

Renormalized Excitonic Model (REM)

Examples:



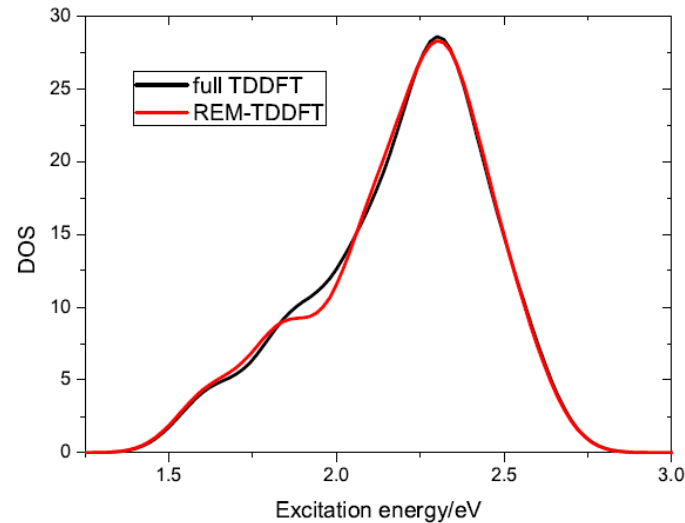
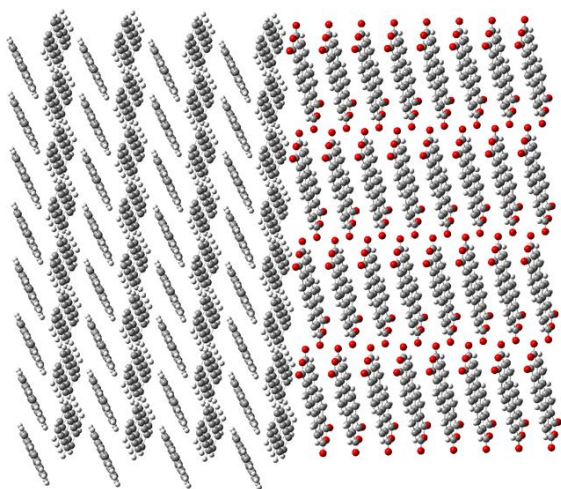
Excitation energy (S_1) in benzene crystal (eV)

system	one-column	two-column	three-column	four-column
REM-TDDFT	5.682	5.664	5.660	5.660
TDDFT	5.685	5.671	5.672	5.672
Δ	-0.003	-0.007	-0.012	-0.012

^aThe LC-BLYP/6-31G is used in the calculations.

Y. Ma, H. Ma, *J. Phys. Chem. A* **2013**, 117, 3655.

DOS for charge transfer states in tetracene/PTCDA interface



H. Ma, A. Troisi, *Adv. Mater.* **2014**, 26, 6163.

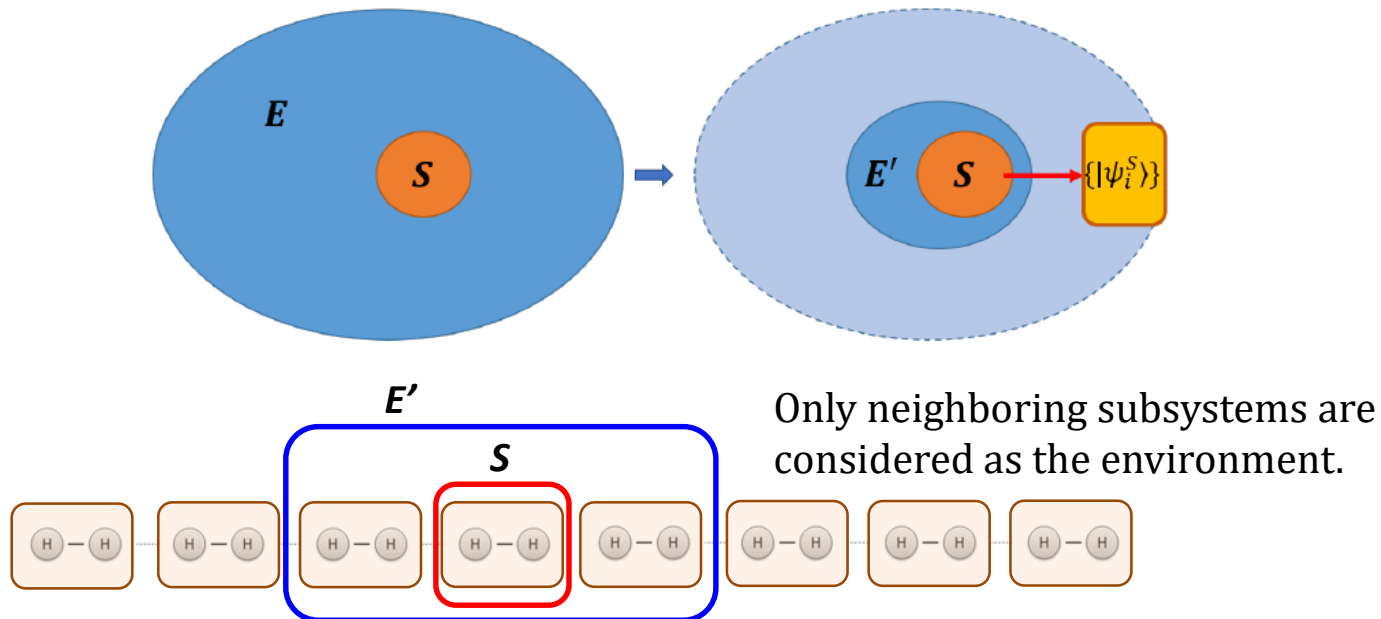
W. Li, H. Ma, S. Li, J. Ma, *Chem. Sci.* **2021**, 12, 14987.

Automatic Construction of Excitonic Basis

Using **quantum information theory (QIT)** to construct excitonic basis (**block interaction product state, BIPS**) **automatically**

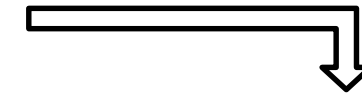
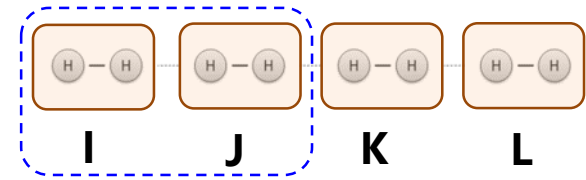
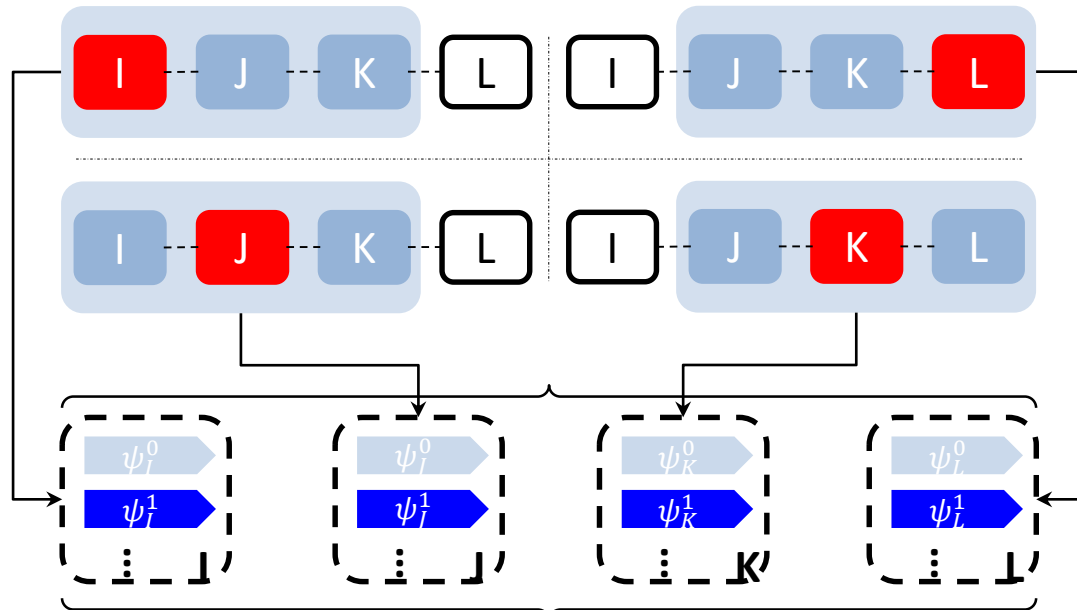
→ Measure system-environment entanglement by the eigenvalues of **reduced density matrix (RDM)**

$$\hat{\rho}_S = \text{Tr}_E |\Psi_{SE}\rangle \langle \Psi_{SE}| = \sum_{e \in \mathcal{H}_E} \langle e | \Psi_{SE}\rangle \langle \Psi_{SE} | e \rangle$$



Automatic Construction of Excitonic Basis

➤ Example: $(H_2)_4$

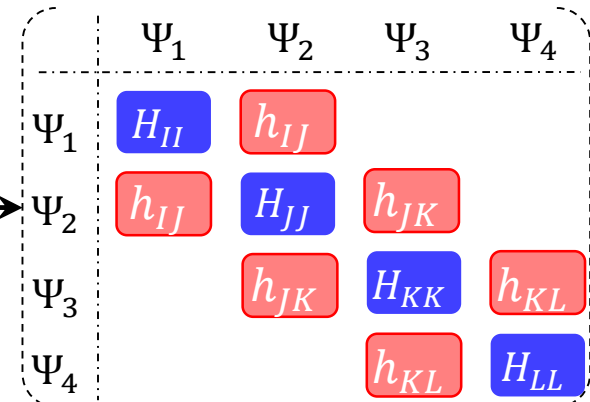
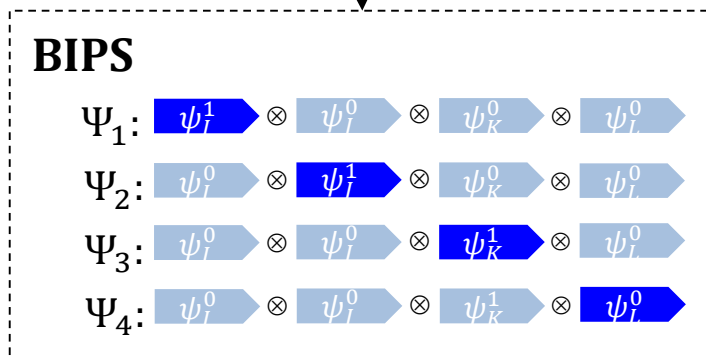


$$\hat{\rho} = |\Psi_{IJK}^*\rangle\langle\Psi_{IJK}^*|$$

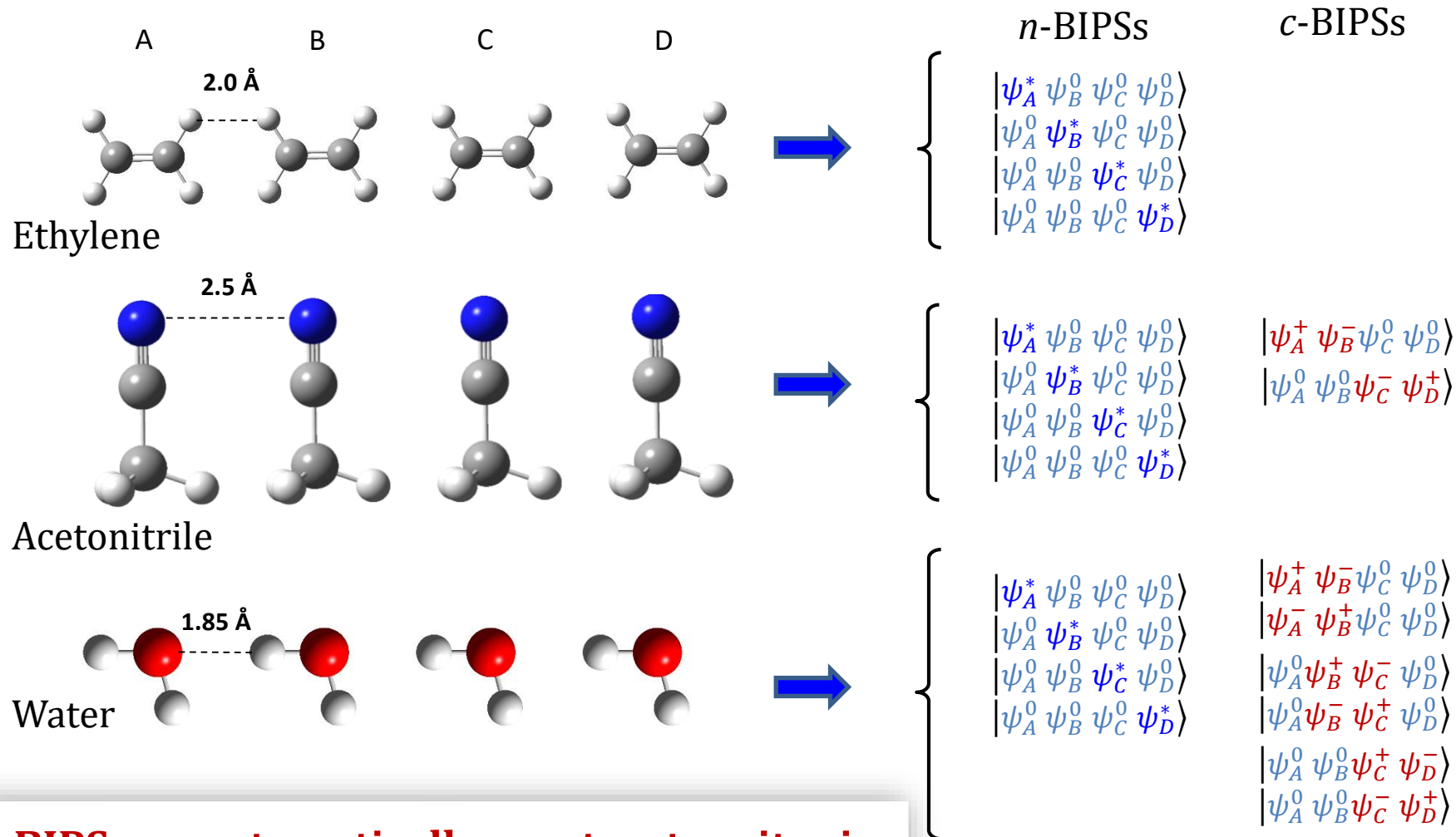
$$\hat{\rho}_J = \text{Tr}_{IK}\hat{\rho} = \sum_{n,m} \rho_{nm}^J |n\rangle\langle m|$$

(Reduce density matrix)

Only consider Two-body effects !



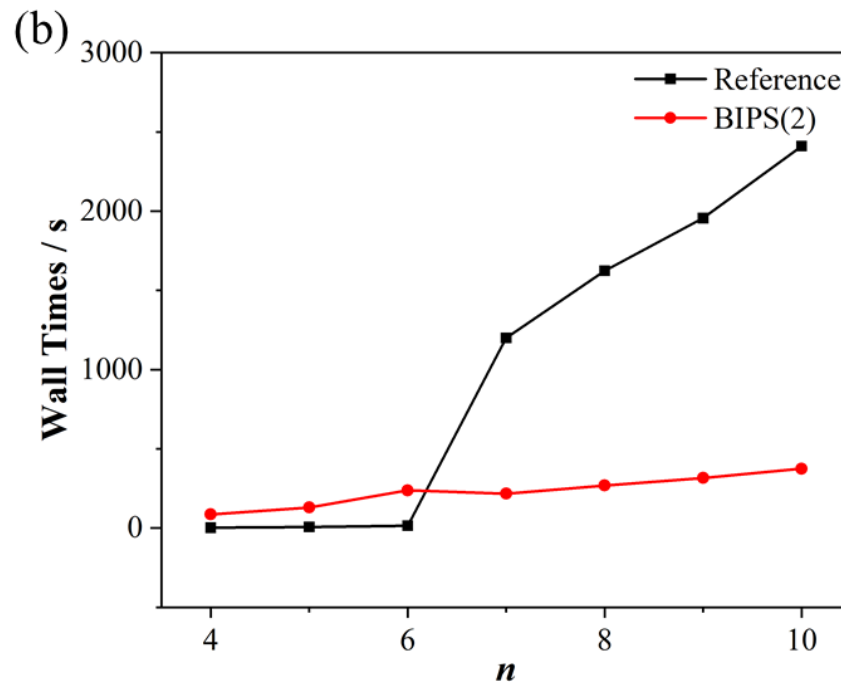
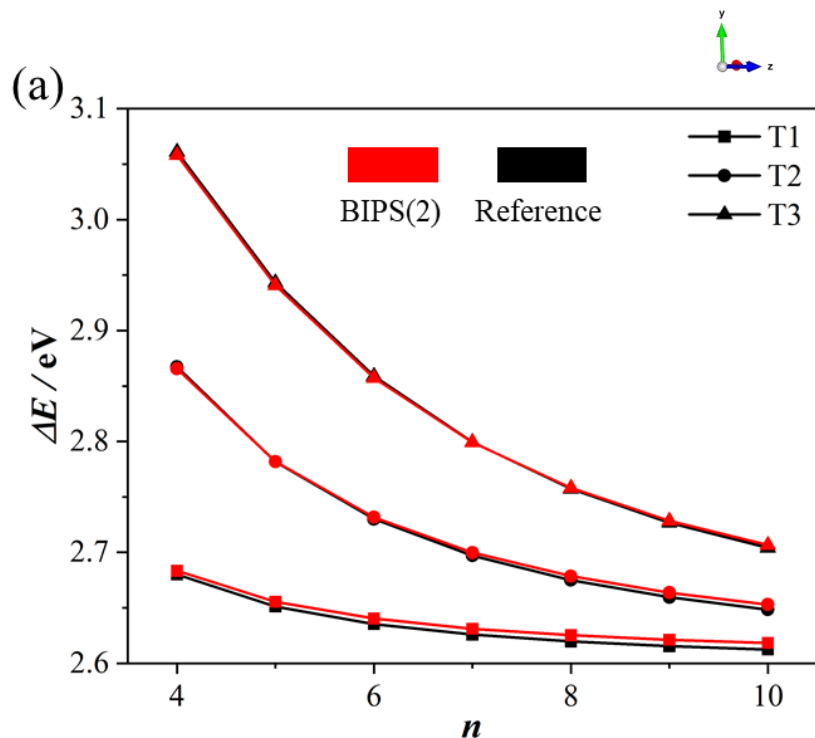
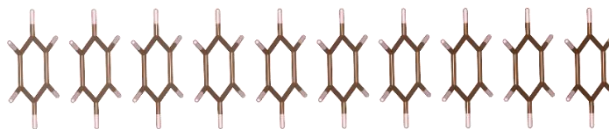
Automatic Construction of Excitonic Basis



BIPS can automatically construct excitonic bases for different aggregated systems.

Automatic Construction of Excitonic Basis

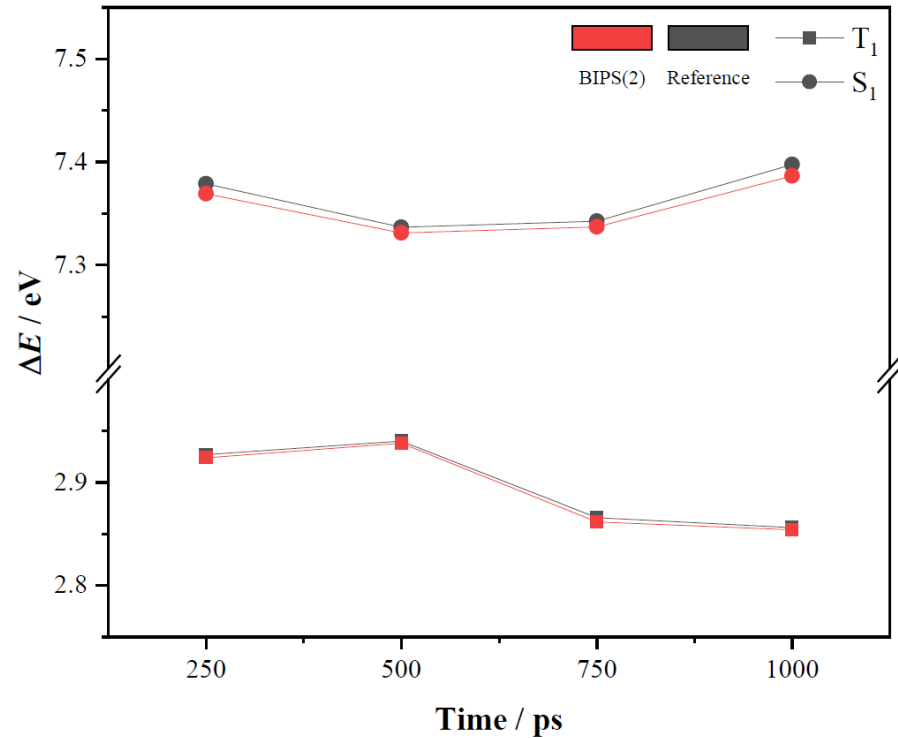
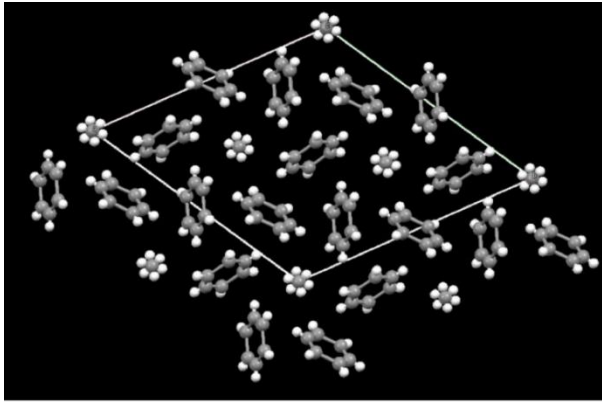
➤ Example: $(C_6H_6)_n$



- BIPS accuracy remains within the maximum deviation of 7 meV.
- BIPS exhibits almost linear growth in time consumption.

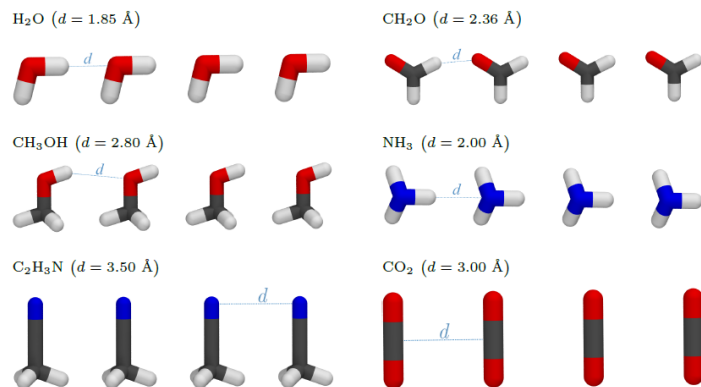
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Automatic Construction of Excitonic Basis



BIPS can correctly mimic the energetic fluctuations under different distorted geometries.

Automatic Construction of Excitonic Basis



$$\mu = \langle \Psi^* | \sum_i q_i \hat{r}_i | \Psi^* \rangle$$

$$\mu^T = \langle \Psi^0 | \sum_i q_i \hat{r}_i | \Psi^* \rangle$$

	$\Delta E / \text{eV}$		$\mu / \text{a.u.}$		$\mu^T / \text{a.u.}$	
	Reference	$\delta(3)$	Reference	$\delta(3)$	Reference	$\delta(3)$
$(\text{H}_2\text{O})_4$	9.1463	0.0112	2.7817	0.0288	0.3091	0.0307
$(\text{CH}_2\text{O})_4$	4.5691	0.0286	4.5822	0.0071	0.0014	0.0006
$(\text{CH}_3\text{OH})_4$	8.4042	0.0094	2.6123	0.0006	0.0415	0.0069
$(\text{NH}_3)_4$	7.9446	0.0120	1.8979	0.0062	0.6309	0.0107
$(\text{C}_2\text{H}_3\text{N})_4$	7.6922	0.0260	5.4202	0.0098	0.0074	0.0033
$(\text{CO}_2)_4$	9.6696	0.0065	0.0001	0.0001	0.0000	0.0000

BIPS provides accurate descriptions for both excitation energies and first-order wave function properties.

Evaluation of Exciton-Phonon Couplings

Direct scanning under linear approximation

$$g_i^I = \frac{\partial \epsilon_i}{\partial Q_I}$$

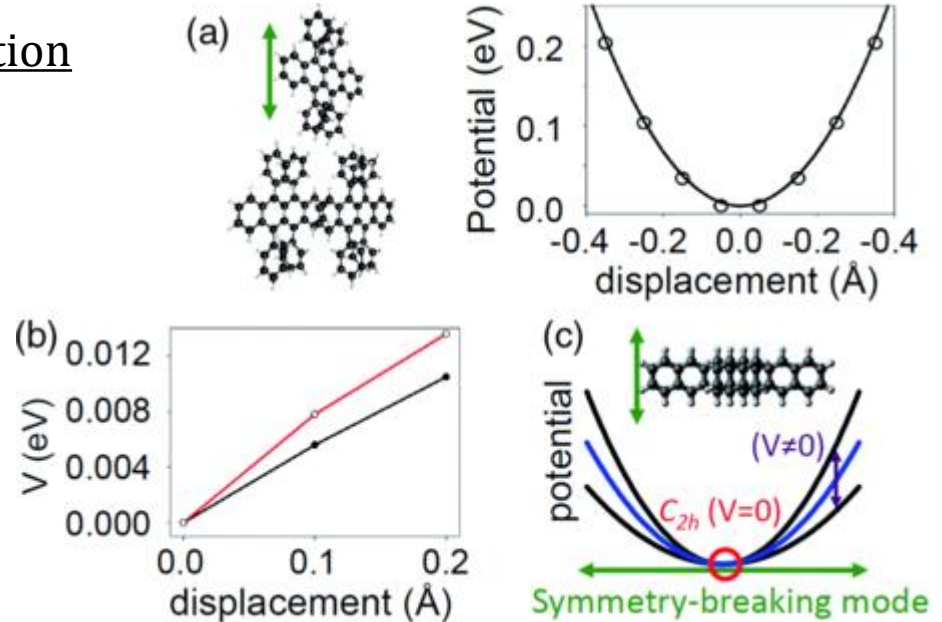
$$g_{ij}^I = \frac{\partial V_{ij}}{\partial Q_I}$$

- Vibration modes in molecule:

$$3N_{\text{atom}} - 6(5)$$

- Phonon modes in crystal:

infinitely many (especially low-frequency ones)



H. Tamura, et al., *Phys. Rev. Lett.* **2015**, 115, 107401.

Due to the **too many phonon modes** in condensed phase, computational costs of direct scanning are **unaffordable!**

Evaluation of Exciton-Phonon Couplings

Use the locality of electronic structure

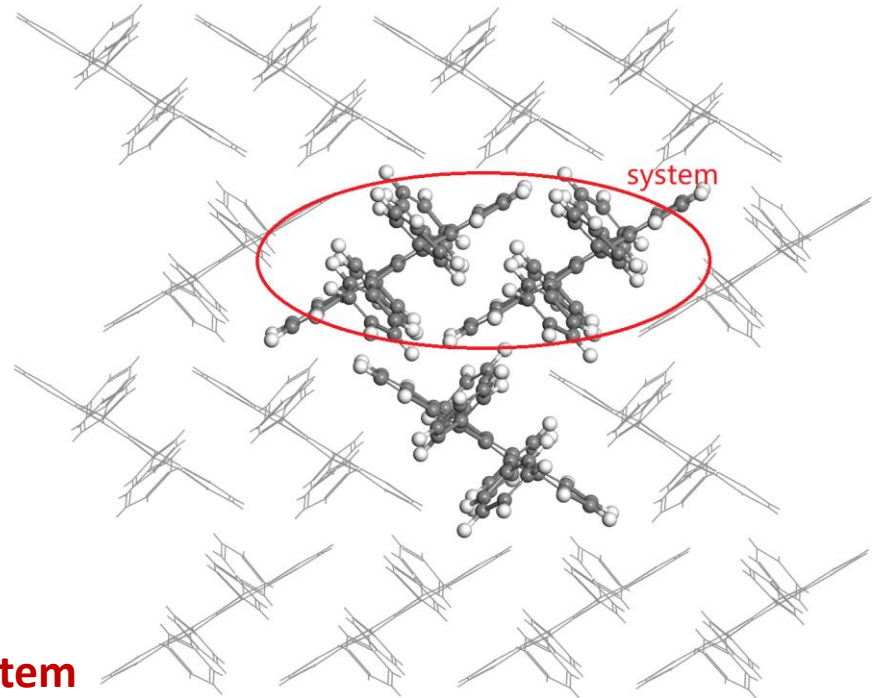
$$g_{ij}^I = \sum_a \frac{\partial V_{ij}}{\partial a} \frac{\partial a}{\partial Q_I} \approx \sum_{a \in \text{system}} \frac{\partial V_{ij}}{\partial a} \frac{\partial a}{\partial Q_I}$$

⇓

$$g_{ij}^I = \nabla V_{ij} \cdot \mathbf{Q}_I^c$$

∇V_{ij} : Quantum chemistry + scanning → **subsystem**

\mathbf{Q}_I^c : phonon band structure → **crystal**



bc plane of rubrene crystal

X. Xie, A. Santana-Bonilla, A. Troisi, *J. Chem. Theory Comput.* **2018**, 14, 3752

Scanning costs: $\infty \rightarrow 3N_{\text{atom}}^{\text{system}}$

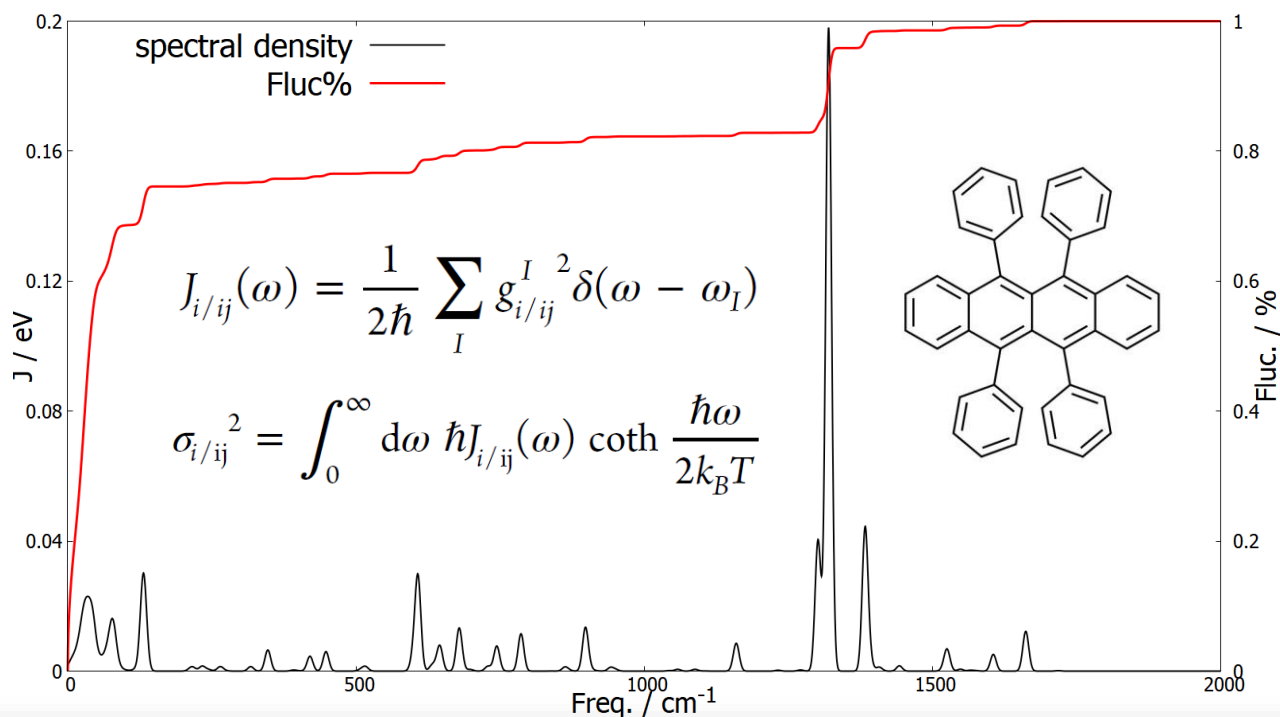
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X. Xie, A. Santana-Bonilla, W. Fang, C. Liu, A. Troisi, H. Ma, *J. Chem. Theory Comput.* **2019**, 15, 3721

Evaluation of Exciton-Phonon Couplings

Example: Non-local ex-ph coupling (CT-TT) of singlet fission (SF) in rubrene

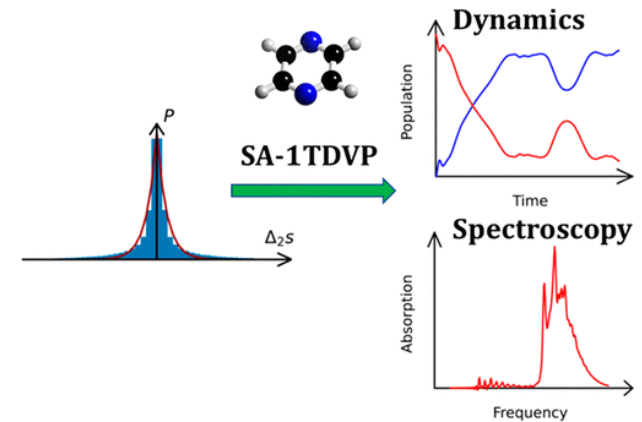
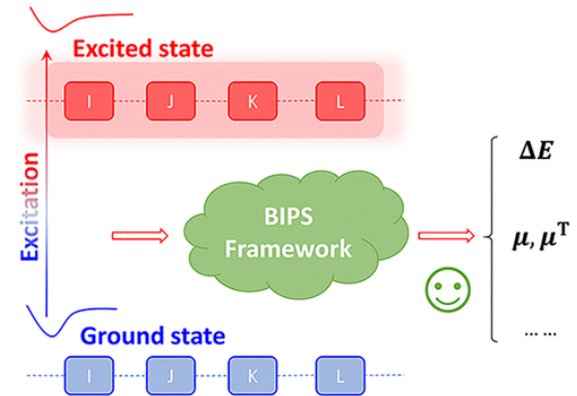
Exp: K. Miyata, et. al. *Nat. Chem.* **2017**, 9, 983.



- 1. Contribution: Low-frequency > High frequency (T= 300 K)**
- 2. Crystal environment is important for SF in rubrene**

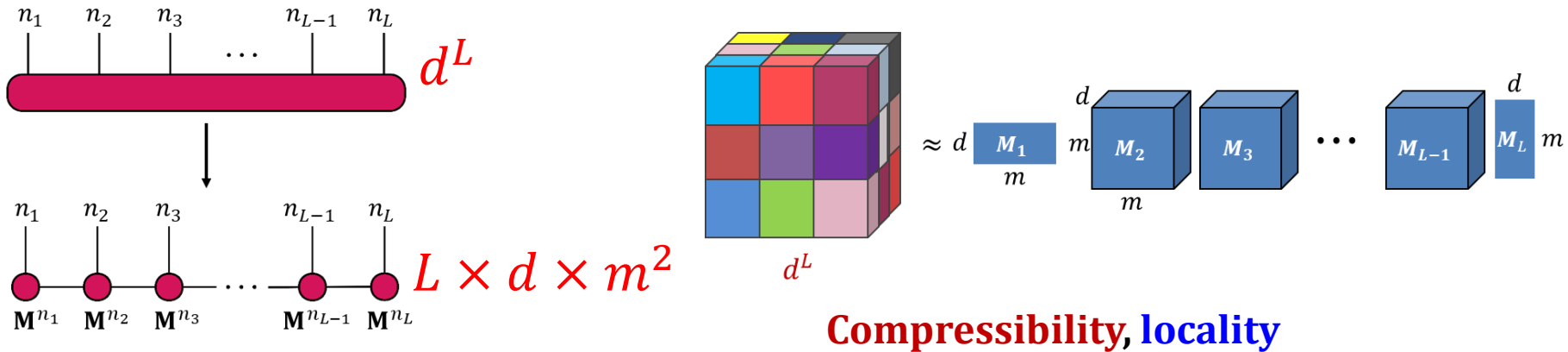
Outline

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Matrix Product State (MPS)

Decomposition of high-rank tensor

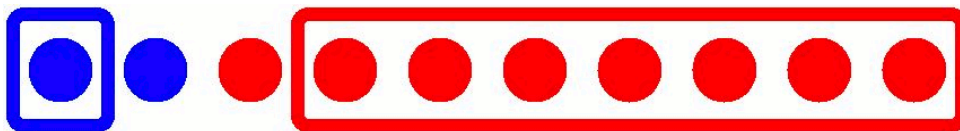


MPS:

$$|\psi\rangle = \sum_{n_1 \dots n_L} \mathbf{M}_1^{n_1} \mathbf{M}_2^{n_2} \dots \mathbf{M}_L^{n_L} |n_1, n_2, \dots, n_L\rangle$$

Density Matrix Renormalization Group (DMRG)

optimize \mathbf{M} matrices iteratively



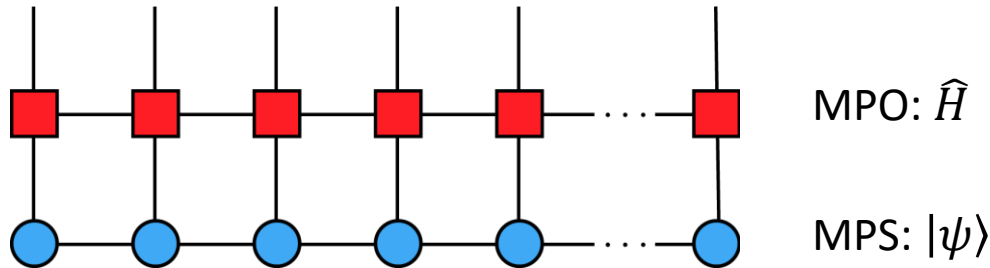
S. White, *Phys. Rev. Lett.* **1992**, 69, 2863.

U. Schollwöck, *Rev. Mod. Phys.* **2005**, 77, 259.

Time-Dependent DMRG (TD-DMRG)

DMRG

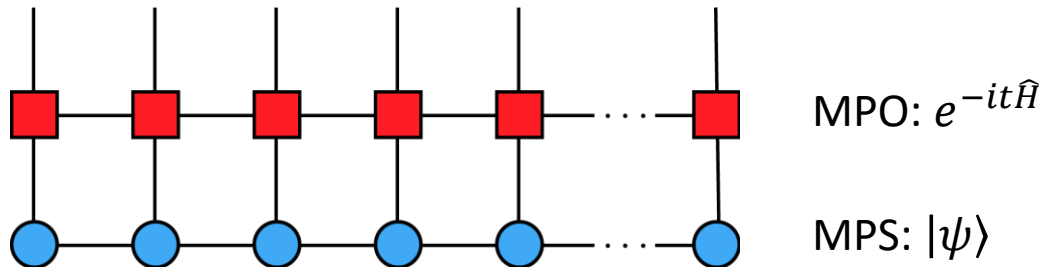
S. White, *Phys. Rev. Lett.* **1992**, 69, 2863.
U. Schollwöck, *Rev. Mod. Phys.* **2005**, 77, 259.



TD-DMRG

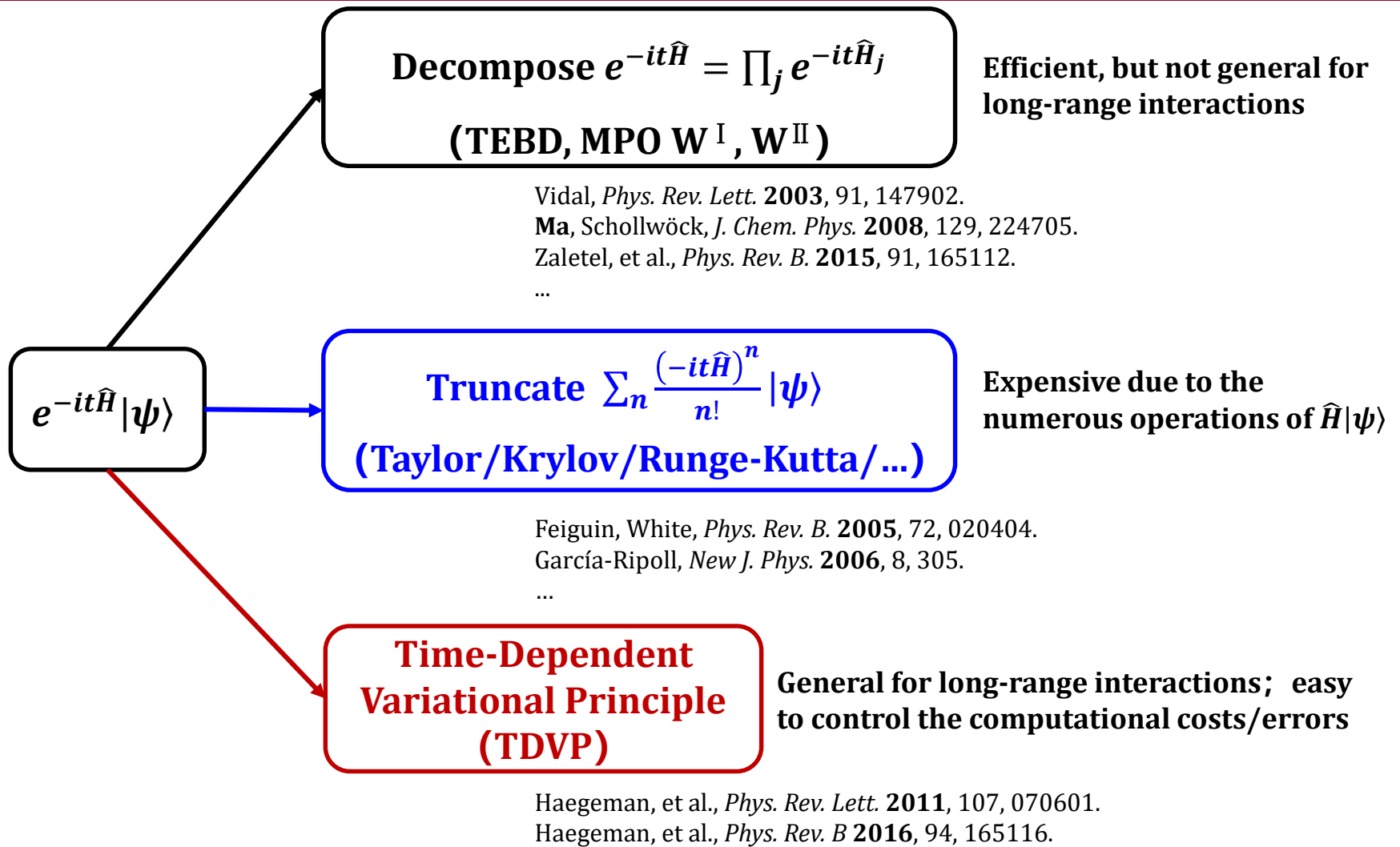


M. A. Cazalilla, J. B. Marston, *Phys. Rev. Lett.* **2002**, 88, 256403.
H. Luo, T. Xiang, X. Wang, *Phys. Rev. Lett.* **2003**, 91, 049701.



How to construct $e^{-it\hat{H}}$ efficiently and accurately?

Time-Dependent DMRG (TD-DMRG)



Reviews: Paeckel, et al., *Ann. Phys.* **2019**, 411, 167998.

Ren, Li, Jiang, Wang, Shuai, *WIREs ComputMol Sci.* **2022**, doi: 10.1002/wcms.1614.

Time-Dependent Variational Principle (TDVP)

Time evolution

$$\hat{H}|\psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle$$

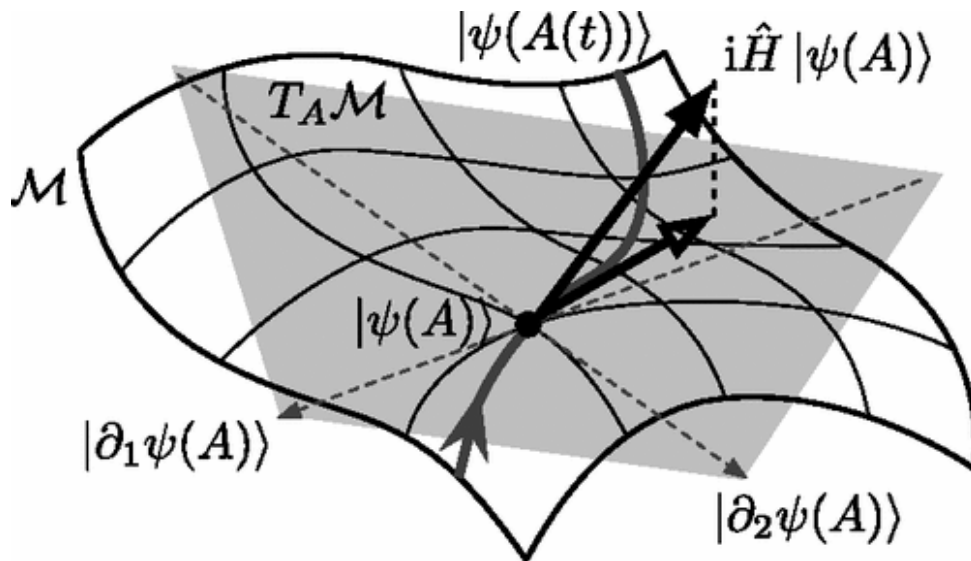


variational optimization

$$\text{Minimize } \|\hat{H}|\psi(t)\rangle - i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle\|^2$$

optimize the basis in which truncated space?

Tangent space



\hat{P}_T is called **tangent space projection operator**, which can project the wave function into the space defined by $\frac{\partial \psi}{\partial M_j}$.

(full: $d^L \rightarrow$ projection: dD^2)

The final evolution function:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{P}_T \hat{H} |\psi(t)\rangle$$

Haegeman, et al., *Phys. Rev. Lett.* **2011**, 107, 070601.

Haegeman, et al., *Phys. Rev. B* **2016**, 94, 165116.

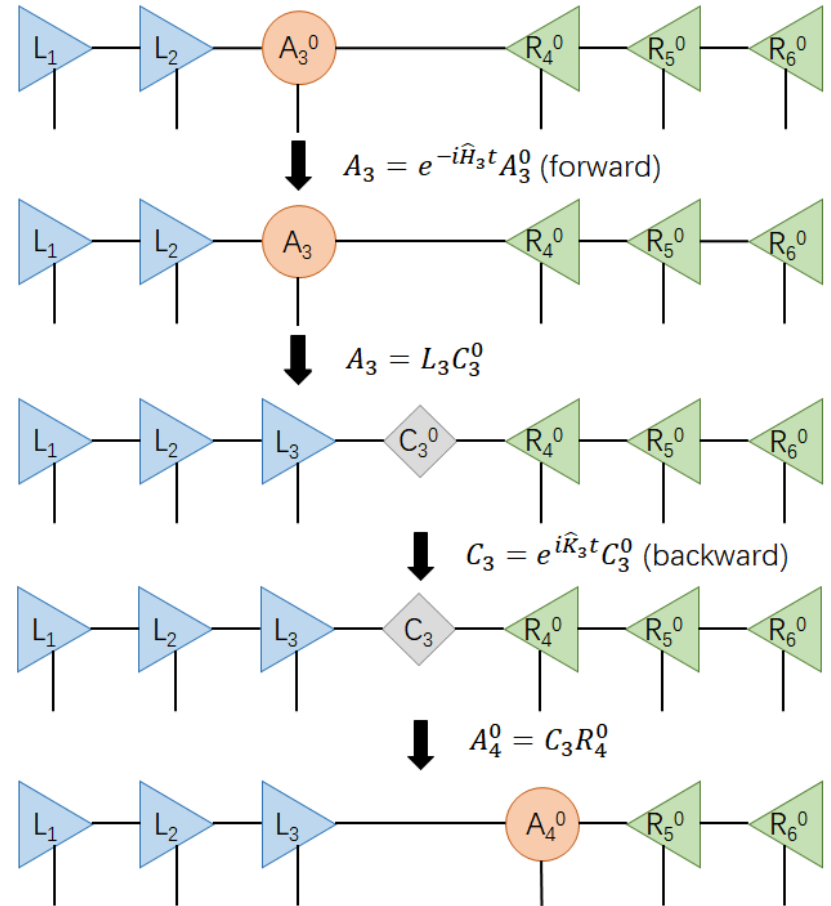
Time-Dependent Variational Principle (TDVP)

\hat{P}_T in the MPS language (**one-site algorithm**):

$$\hat{P}_T = \sum_{j=1}^n \hat{P}_{j-1}^L \otimes \hat{I}_j \otimes \hat{P}_{j+1}^R - \sum_{j=1}^{n-1} \hat{P}_j^L \otimes \hat{P}_{j+1}^R$$

Single time step

= **n steps of forward evolution**
 + **$n-1$ steps of backward evolution**



$$|\psi\rangle = \sum_{n_1 \dots n_L} \mathbf{M}_1^{n_1} \mathbf{M}_2^{n_2} \dots \mathbf{M}_L^{n_L} |n_1, n_2, \dots, n_L\rangle$$

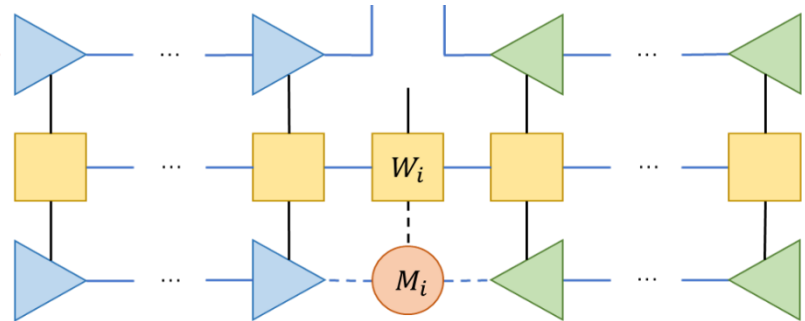
Single-Site vs. Two-Site

\hat{P}_T (two-site algorithm)

$$\hat{P}_T = \sum_{j=1}^{n-1} \hat{P}_{j-1}^L \otimes \hat{I}_j \otimes \hat{I}_{j+1} \otimes \hat{P}_{j+2}^R - \sum_{j=1}^{n-2} \hat{P}_j^L \otimes \hat{I}_{j+1} \otimes \hat{P}_{j+2}^R$$

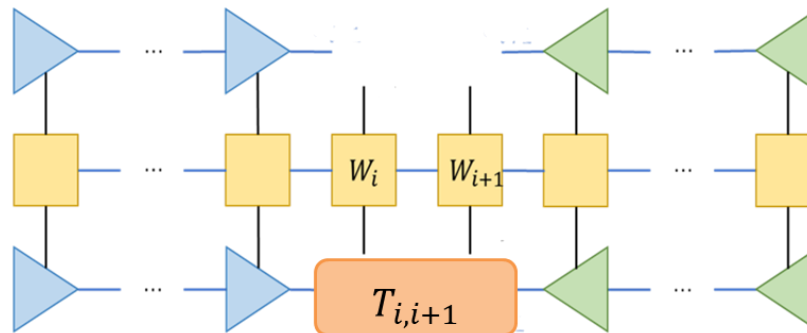
1TDVP:

update one site
at each step



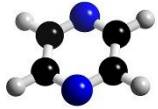
2TDVP:

update two sites
at each step



1TDVP vs. 2TDVP

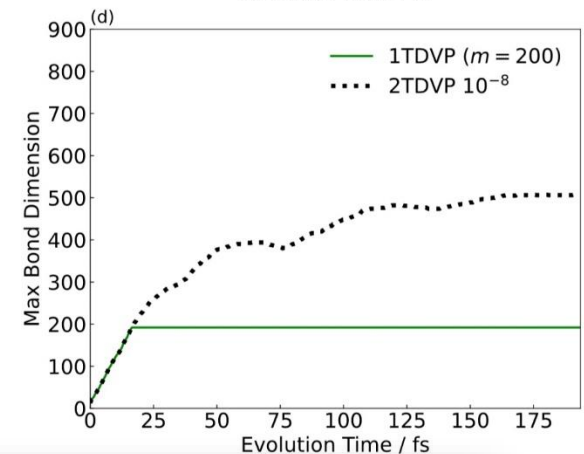
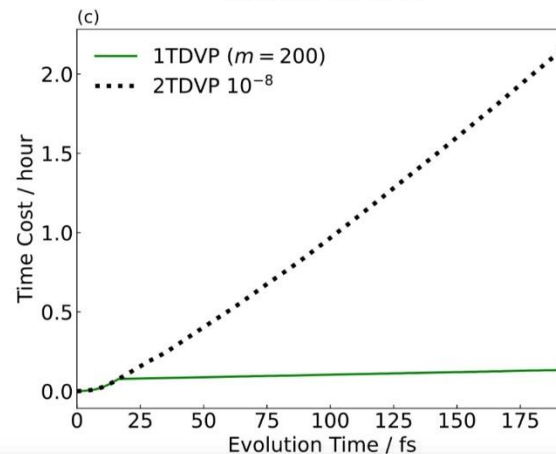
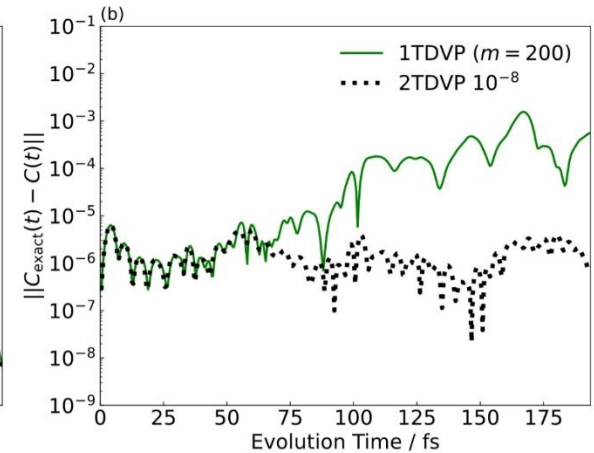
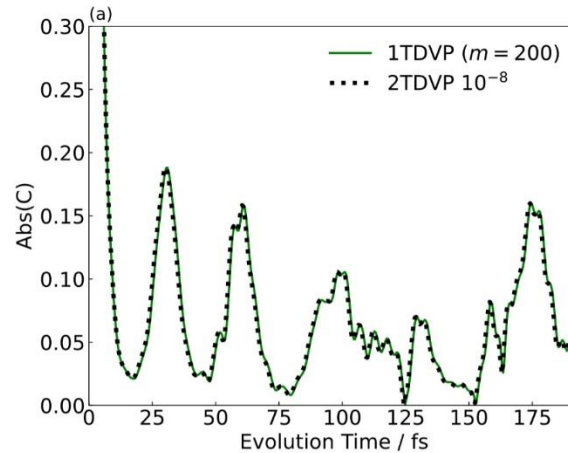
Test of efficiency and accuracy (model: the 4-mode pyrazine)



$$\hat{H}_{\text{ex}} = \sum_i \varepsilon_i \hat{c}_i^\dagger \hat{c}_i + \sum_{i \neq j} V_{ij} \hat{c}_i^\dagger \hat{c}_j$$

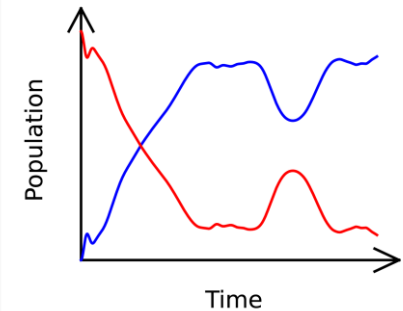
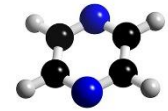
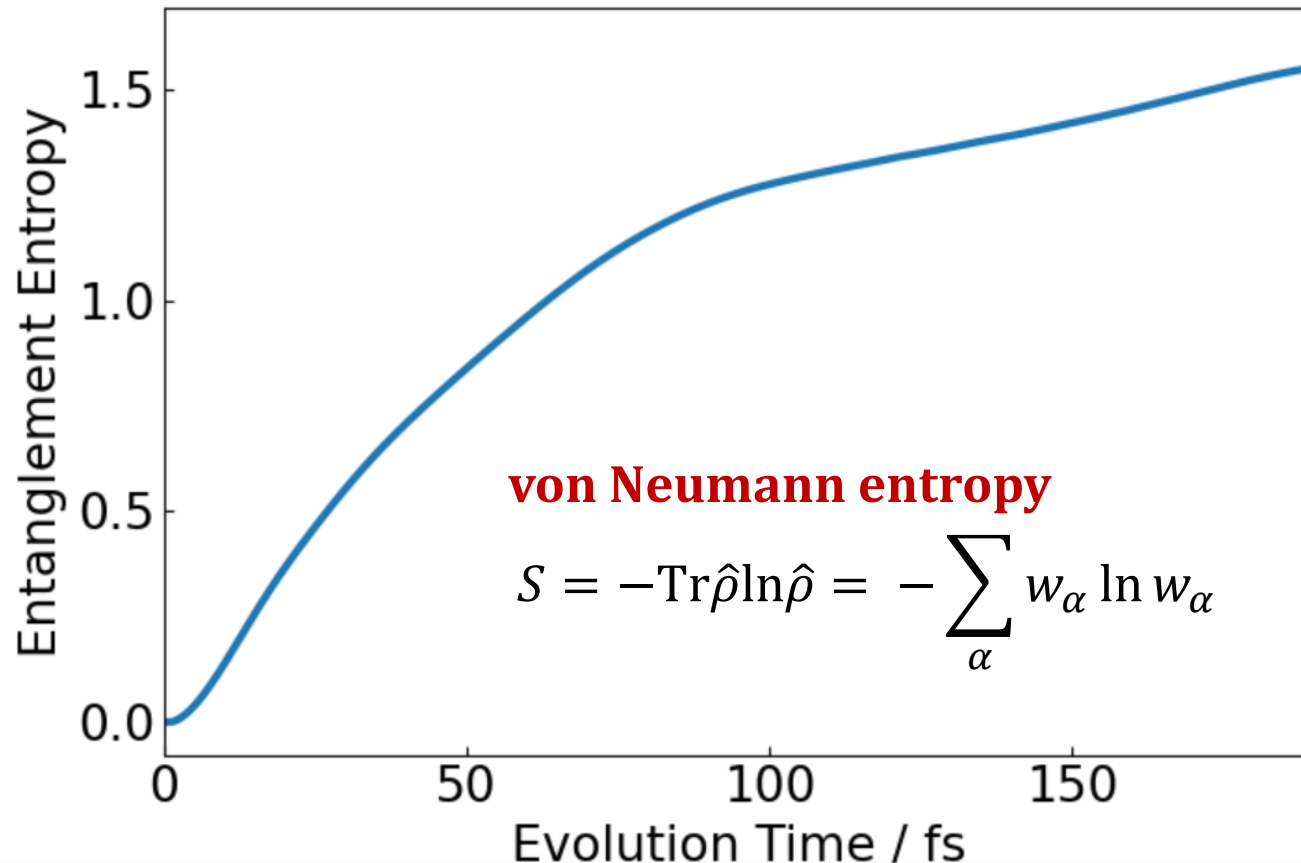
$$\hat{H}_{\text{vib}} = \sum_{I=1}^{N_{\text{vib}}} \omega_I \hat{b}_I^\dagger \hat{b}_I$$

$$\hat{H}_{\text{ex-vib}} = \sum_{i,j,I} g_{ij,I}^I \hat{c}_i^\dagger \hat{c}_j (\hat{b}_I^\dagger + \hat{b}_I)$$



1TDVP is faster, but its error is uncontrollable at long time limit.

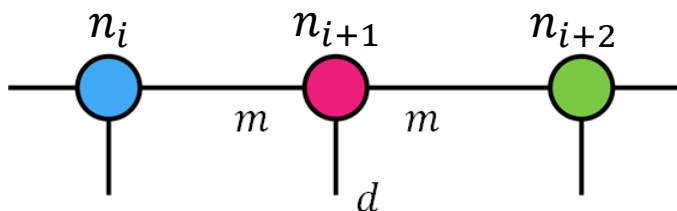
Entanglement Growth during Time Evolution



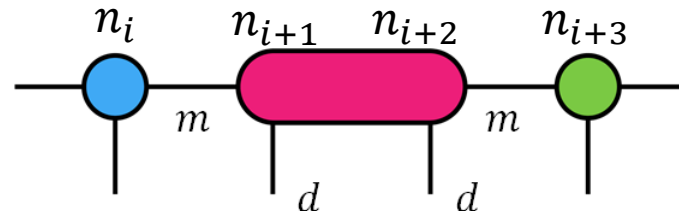
The entanglement grows as time evolves, so the adaptive increasing of m in MPS is needed.

1TDVP vs 2TDVP

1TDVP



2TDVP



Variational space	$m \times d \times m$	$m \times d \times d \times m$
Efficiency	high	low
Increase m ?	No (m non-zero singular values at most)	Yes ($m \times d$ non-zero singular values at most)

(1)



(2) How to set the m or make m increase automatically in 1TDVP?

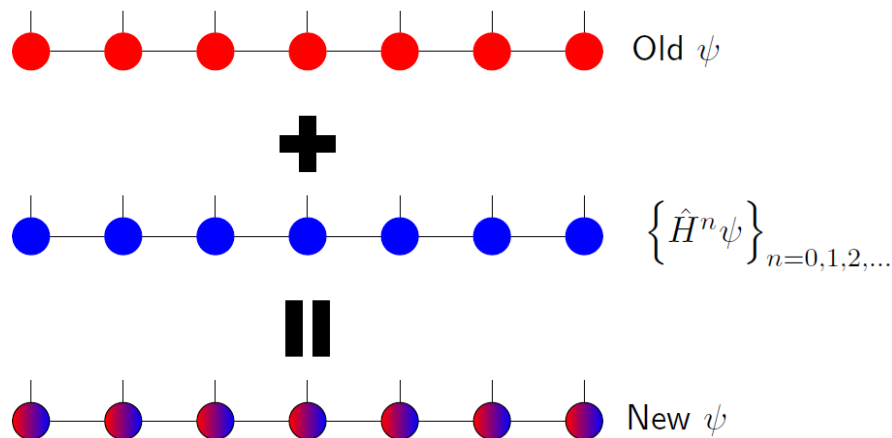
Xie, Liu, Yao, Schollwöck, Liu, **Ma**, *J. Chem. Phys.* **2019**, 151, 224101.

New adaptive 1TDVP method is needed.

Adaptive 1TDVP

Global treatment

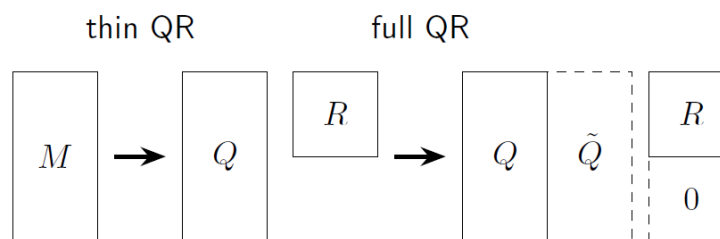
Yang, White, *Phys. Rev. B*
2020, 102, 094315.



Extend the space through MPS addition

Local treatment

Dunnett, Chin, *Phys. Rev. B*
2021, 104, 214302.



Extend the space through full QR and converge by calculating the measure function

$$\text{Measure function: } f_j = \|H_{\text{eff},j}A_j\|^2 - \|K_{\text{eff},j}C_j\|^2 + \|H_{\text{eff},j+1}M_{j+1}\|^2$$

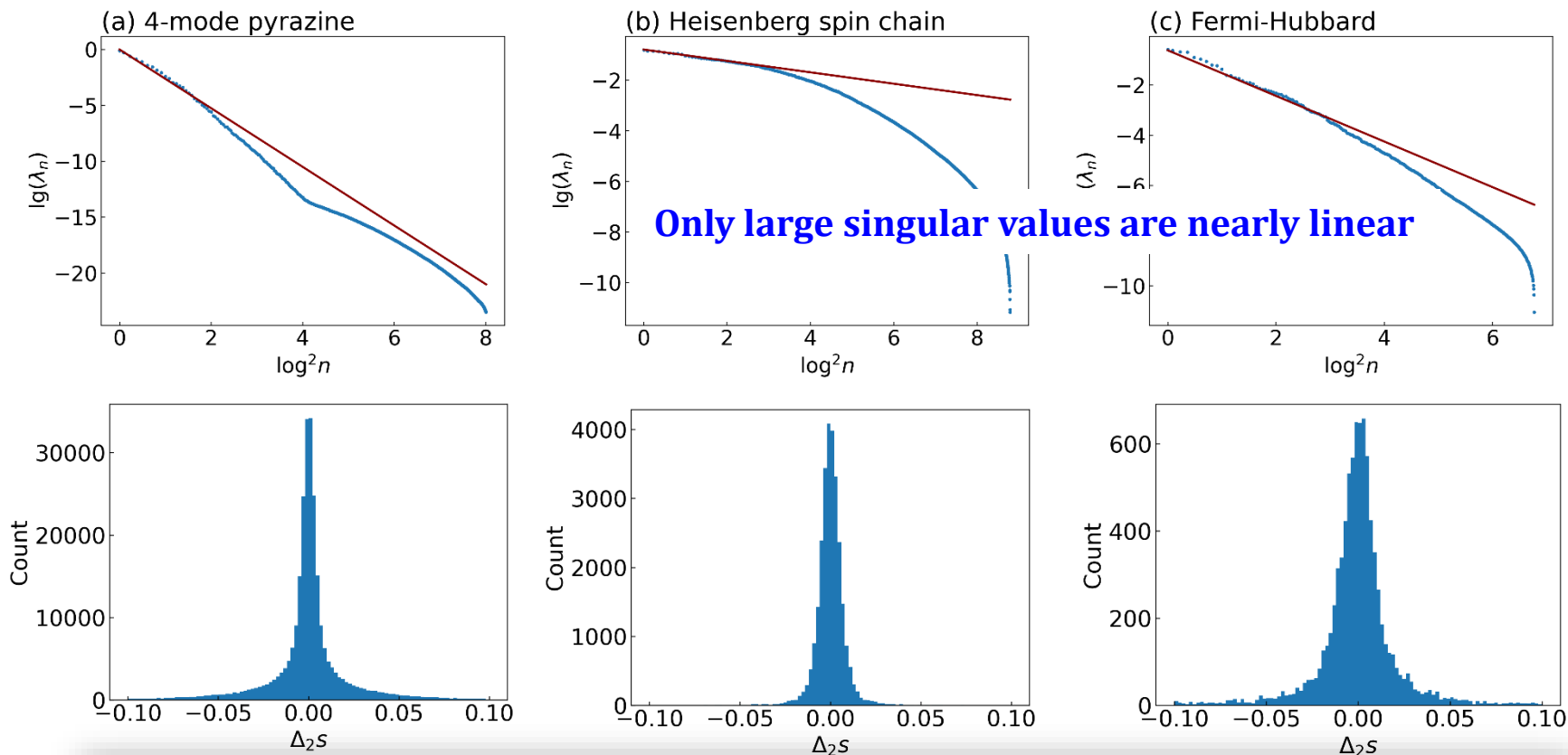
Both need additional expensive $\hat{H}|\psi\rangle$ operations.

Statistical Distribution of Singular Values in TD-DMRG

$$\lambda_n \sim \exp(\text{const} \times \ln^2 n)$$

Okunishi, Hieida, Akutsu, *Phys. Rev. E* **1999**, 59, R6227.

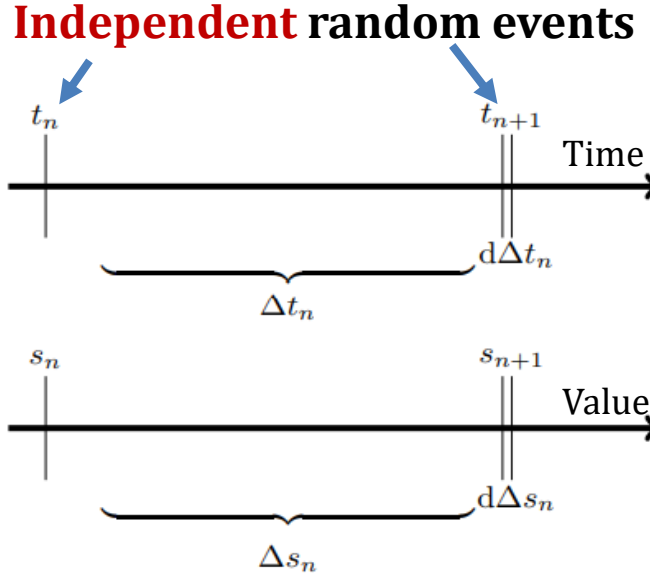
Chan, Ayers, Croot, *J. Stat. Phys.* **2002**, 109, 289.



Distributions of 2nd order singular value logarithms show similar features in different models. (exponential? Gaussian?)

Derivation of exponential distribution

Poisson process:



Distribution of singular value logarithms ($s_n = \log \lambda_n$):

$$P(\Delta s_n) = \beta e^{-\beta \Delta s_n}$$

$$P(\Delta s_{n+1}) = \beta e^{-\beta \Delta s_{n+1}}$$

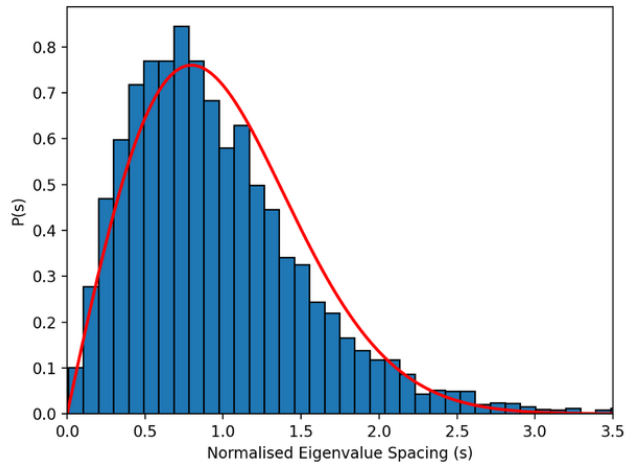
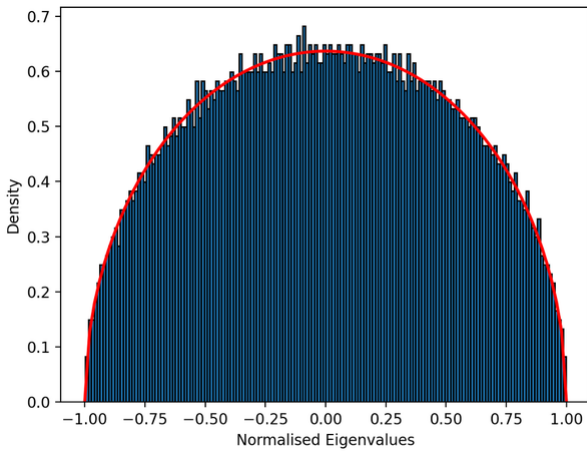
For $\Delta_2 s = \Delta s_{n+1} - \Delta s_n$

$$\begin{aligned} P(\Delta_2 s) d(\Delta_2 s) &= \int_0^{+\infty} d(\Delta s_n) \int_0^{+\infty} d(\Delta s_{n+1}) \beta^2 e^{-\beta \Delta s_n} e^{-\beta \Delta s_{n+1}} \\ &= \int_{\max(0, \Delta_2 s)}^{+\infty} d(\Delta s_n) \beta^2 e^{-\beta \Delta s_n} e^{-\beta(\Delta_2 s + \Delta s_n)} d(\Delta_2 s) \end{aligned}$$



$$P(\Delta_2 s) = \frac{\beta}{2} \exp(-\beta |\Delta_2 s|)$$

Random Matrix Theory (RMT): **Coupled** random events



Random matrix

distribution of eigenvalues

distribution of level spacings

Wigner surmise: $P(\Delta s) = \Delta s e^{-\frac{1}{2}(\Delta s)^2}$



E. P. Wigner, *Ann. Math.* **1955**, 62, 548; **1958**, 67, 325.

Derivation of Quasi-Gaussian distribution

For level spacing Δs_n in RMT, **Wigner surmise** gives:

$$P(\Delta s_n) = \beta \Delta s_n e^{-\frac{\beta}{2}(\Delta s_n)^2}$$

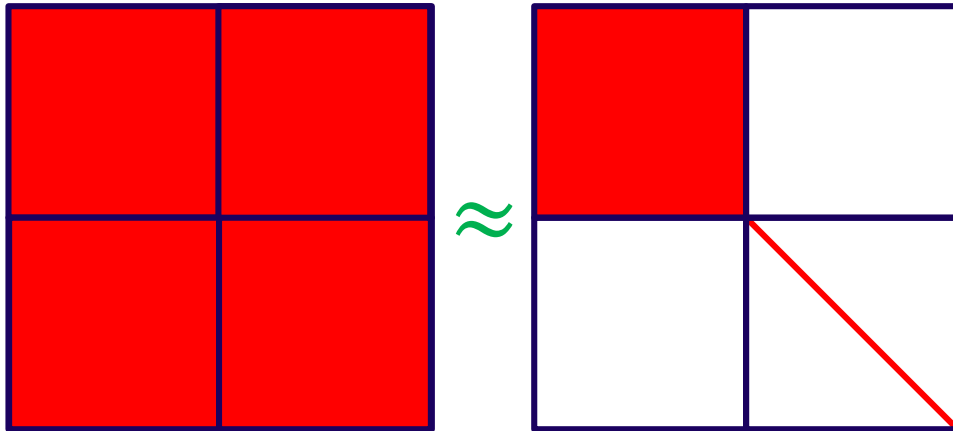
one can derive the distribution of 2nd order gradients:

$$\begin{aligned} P(\Delta_2 s) d(\Delta_2 s) &= \beta^2 \int_0^{+\infty} d(\Delta s_n) \int_0^{+\infty} d(\Delta s_{n+1}) \Delta s_n \Delta s_{n+1} e^{-\frac{\beta}{2}(\Delta s_n)^2} e^{-\frac{\beta}{2}(\Delta s_{n+1})^2} \\ &= \beta^2 d(\Delta_2 s) \int_0^{+\infty} d(\Delta s_n) \Delta s_n (\Delta s_n + \Delta_2 s) e^{-\frac{\beta}{2}(\Delta s_n)^2} e^{-\frac{\beta}{2}(\Delta s_n + \Delta_2 s)^2} \\ &= \beta^2 d(\Delta_2 s) e^{-\frac{\beta}{4}(\Delta_2 s)^2} \int_{\frac{\Delta_2 s}{2}}^{+\infty} dy \left[y^2 - \left(\frac{\Delta_2 s}{2} \right)^2 \right] e^{-\beta y^2} \end{aligned}$$

$$P(\Delta_2 s) = \beta^2 e^{-\frac{\beta}{4}(\Delta_2 s)^2} \int_{\frac{\Delta_2 s}{2}}^{+\infty} dy \left[y^2 - \left(\frac{\Delta_2 s}{2} \right)^2 \right] e^{-\beta y^2}$$

Stochastic Adaptive 1TDVP (SA-1TDVP)

Idea:



Matrix diagonalization (large) \approx Matrix diagonalization (small) + prediction of other small eigenvalues by stochastic methods

Poisson distribution

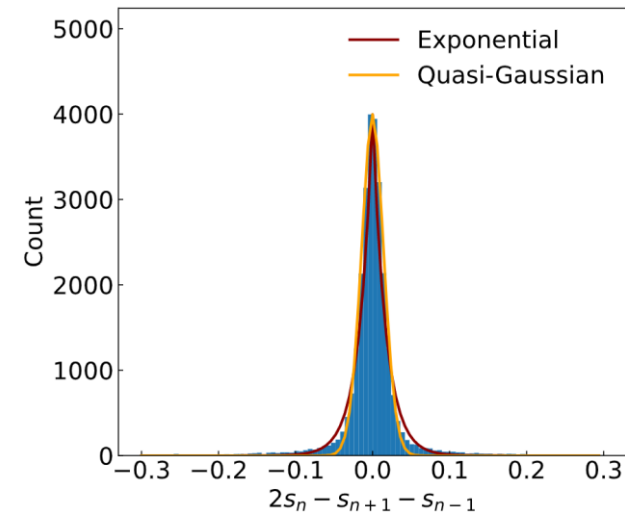
(**Independent** random events)

$$P(\Delta s_n) = \beta e^{-\beta \Delta s_n}$$

Wigner surmise

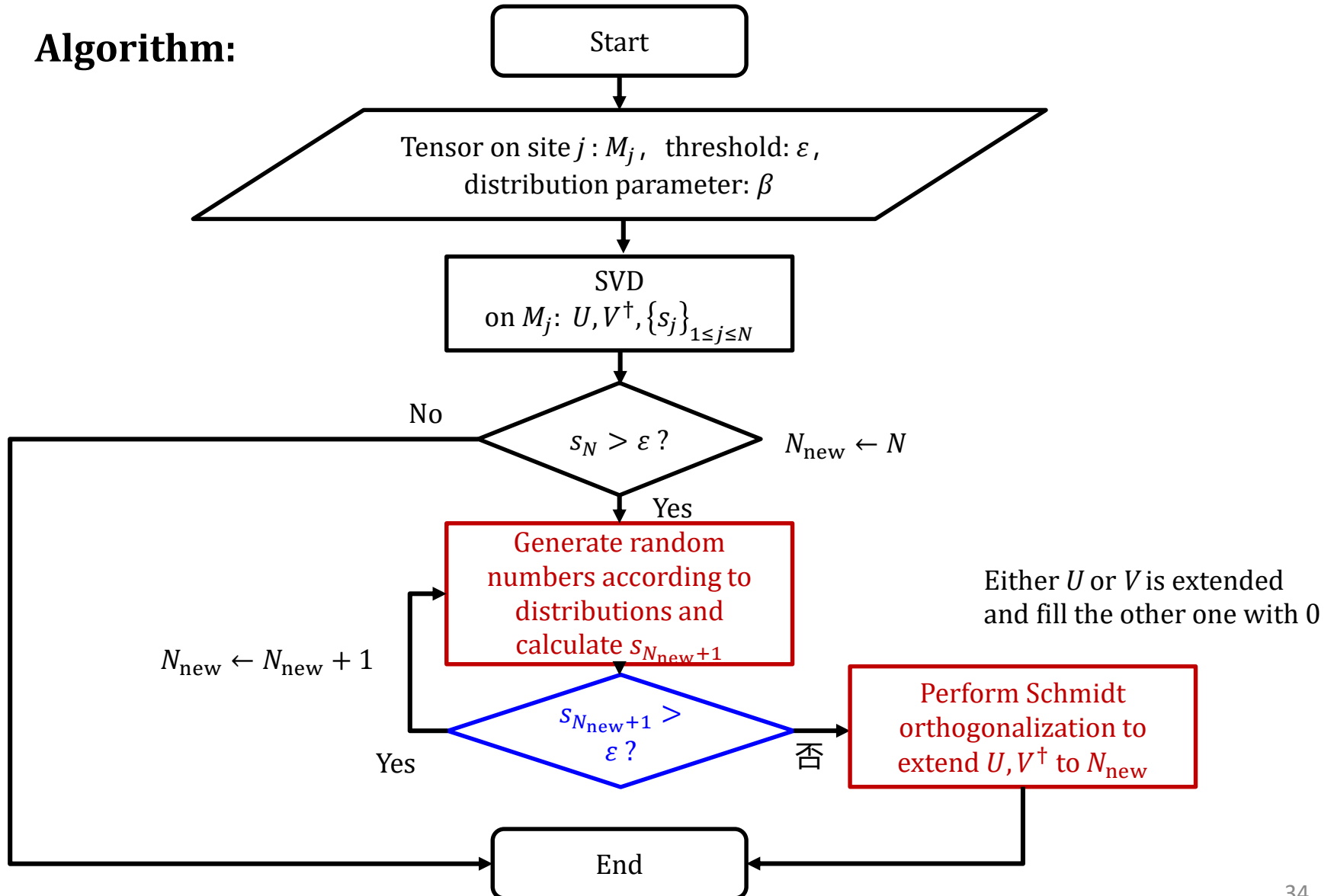
(**Coupled** random events)

$$P(\Delta s_n) = \beta \Delta s_n e^{-\frac{\beta}{2}(\Delta s_n)^2}$$



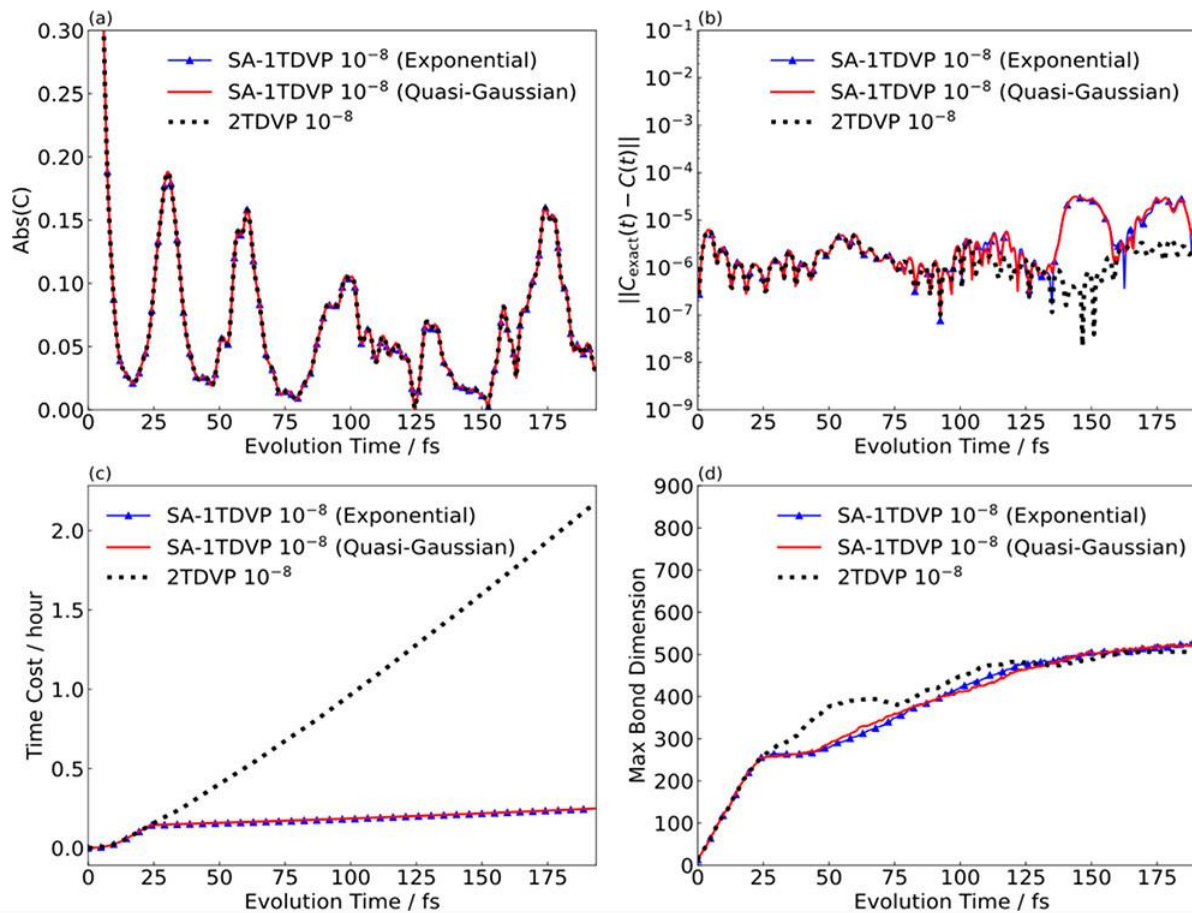
Stochastic Adaptive 1TDVP (SA-1TDVP)

Algorithm:



SA-1TDVP

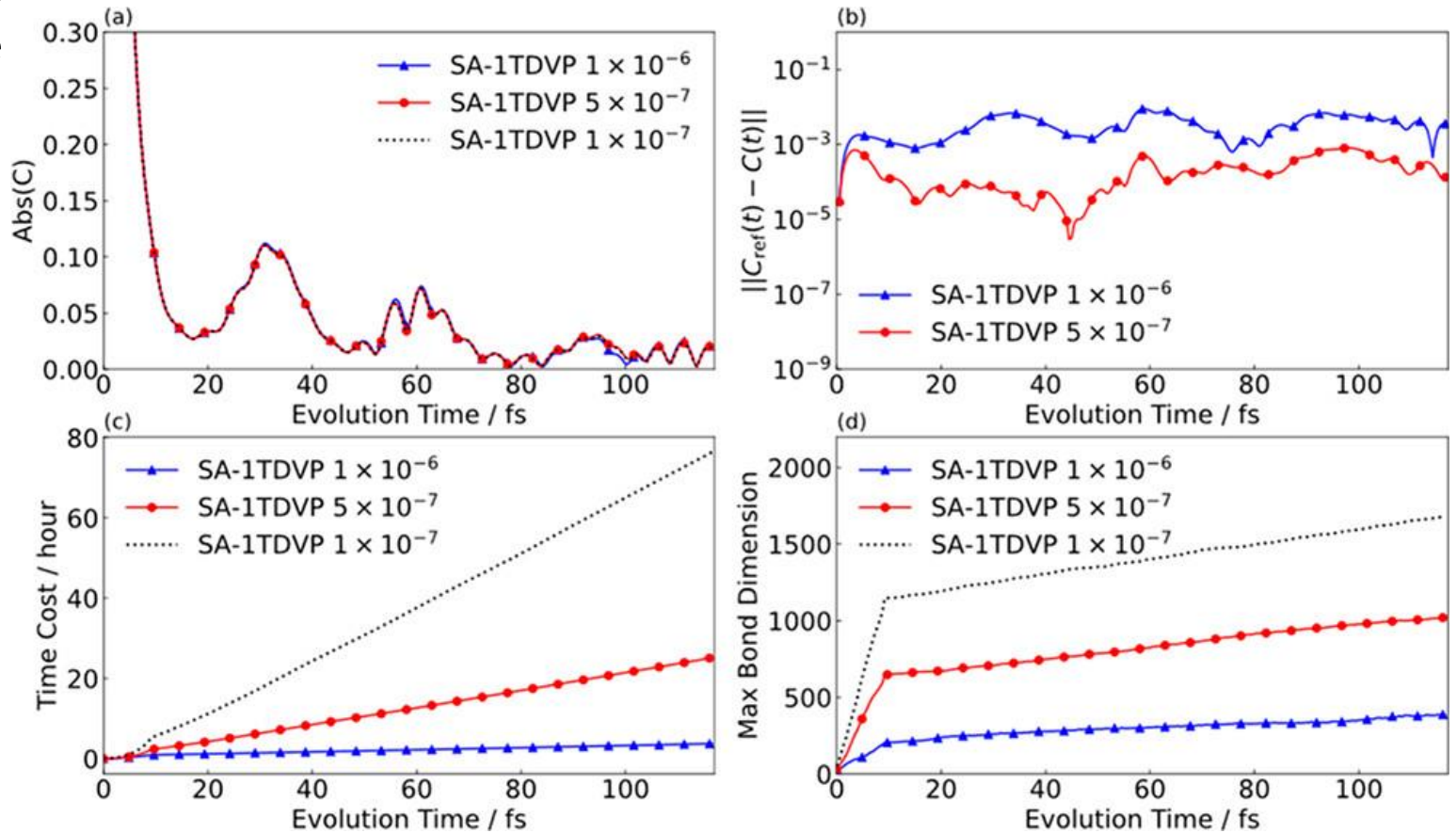
4-mode pyrazine



Two distributions for SA-1TDVP give similar performances and reduce ~80% computational time of 2TDVP.

SA-1TDVP

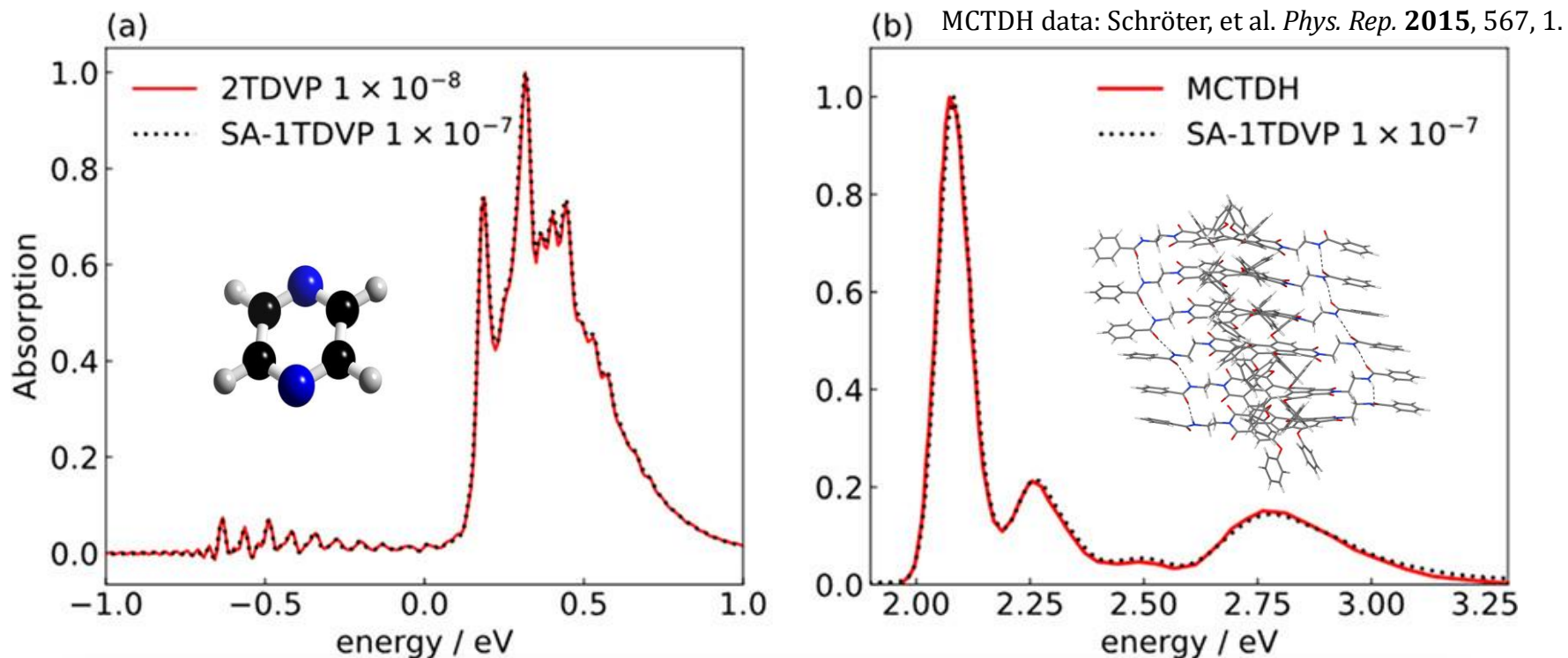
24-mode pyrazine



SA-1TDVP's error does not increase significantly as time evolves

SA-1TDVP

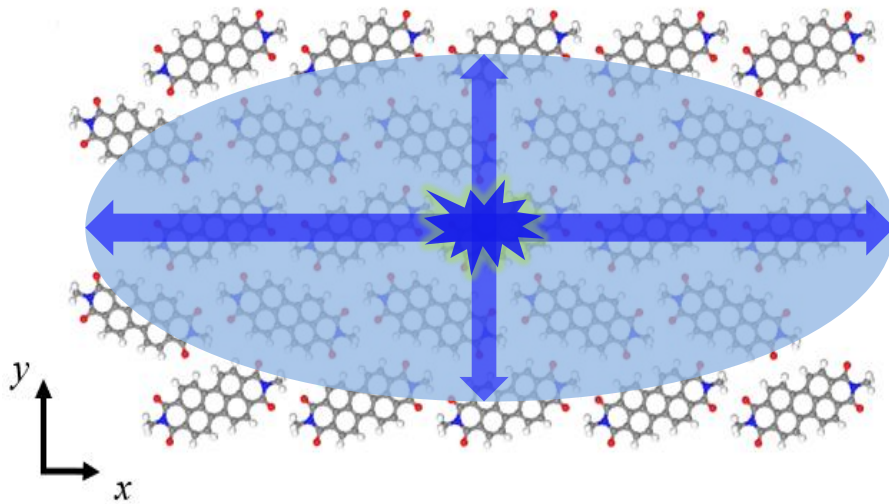
Simulated spectra of pyrazine and PBI



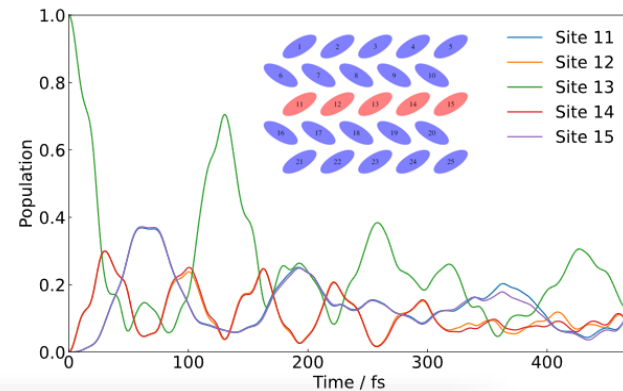
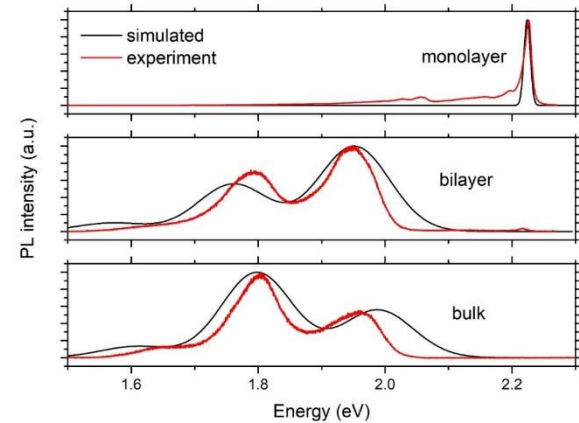
SA-1TDVP can serve as an efficient and automatic tool for quantum dynamic and spectroscopy simulations.

SA-1TDVP

Application: exciton diffusion in 2D molecular crystal of PTCDI



Zhao, **Ma**, Javey, Wang, et al., *Nat. Commun.* **2019**, 10, 5589.



Strong anisotropy and J-aggregation effect

Xu, Xie, Xie, Schollwöck, **Ma**, *JACS Au* **2022**, 2, 335.

Summary

➤ BIPS provides an automatic and efficient scheme for the **accurate construction of the bases for excitonic models.**



Model construction

➤ **Ex-ph couplings** can be quantitatively evaluated by utilizing the locality of the electronic structures.



Parameter evaluation

➤ SA-1TDVP can **capture the entanglement growth adaptively and efficiently** in long-time simulations.



Quantum dynamics

Acknowledgement

Group members in NJU:



Ke Wang,



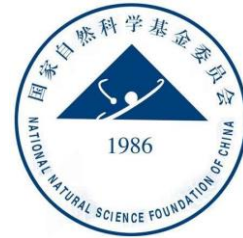
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(LMU Munich)



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(Univ. Liverpool)



Thanks for your kind attention!