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# Branching Correction for Mixed Quantum-Classical Dynamics

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#### **Education and Research Experiences**



















- Introduction to Mixed Quantum-Classical Dynamics
- Interpretation of Decoherence Correction
- Branching Corrected Surface Hopping
- New Energy-Based Decoherence Time Formulas
- Branching Corrected Mean Field





#### Introduction to Mixed Quantum-Classical Dynamics

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### **Born-Oppenheimer Approximation**



# Schrödinger equation: $\mathbf{H}\psi_{n}(\{\vec{R}_{\alpha},\vec{r}_{i}\}) = E_{n}\psi_{n}(\{\vec{R}_{\alpha},\vec{r}_{i}\})$ $\mathbf{H} = \sum_{\alpha} \frac{-\nabla_{\alpha}^{2}}{2M_{\alpha}} + \sum_{i} \frac{-\nabla_{i}^{2}}{2} + \sum_{\alpha < \beta} \frac{Z_{\alpha}Z_{\beta}}{\left|\vec{R}_{\alpha} - \vec{R}_{\beta}\right|} + \sum_{i < j} \frac{1}{\left|\vec{r}_{i} - \vec{r}_{j}\right|} + \sum_{\alpha,i} \frac{-Z_{\alpha}}{\left|\vec{r}_{i} - \vec{R}_{\alpha}\right|}$



#### Born-Oppenheimer approximation:

$$\begin{split} \mathbf{H} &= \sum_{\alpha} \frac{-\nabla_{\alpha}^{2}}{2M_{\alpha}} + \mathbf{H}_{e} \\ \mathbf{H}_{e} &= \sum_{i} \frac{-\nabla_{i}^{2}}{2} + \sum_{\alpha < \beta} \frac{Z_{\alpha} Z_{\beta}}{\left|\vec{R}_{\alpha} - \vec{R}_{\beta}\right|} + \sum_{i < j} \frac{1}{\left|\vec{r}_{i} - \vec{r}_{j}\right|} + \sum_{\alpha, i} \frac{-Z_{\alpha}}{\left|\vec{r}_{i} - \vec{R}_{\alpha}\right|} \\ \mathbf{H}_{e}(\{\vec{R}_{\alpha}\}) \psi_{n}(\{\vec{r}_{i}; \vec{R}_{\alpha}\}) = E_{n}(\{\vec{R}_{\alpha}\}) \psi_{n}(\{\vec{r}_{i}; \vec{R}_{\alpha}\}) \end{split}$$



### **Adiabatic and Nonadiabatic Dynamics**





In nonadiabatic dynamics, electron and nuclear dynamics are strongly coupled

### Nonadiabatic Dynamics in Photochemistry and Thermochemistry





#### **Photoisomerization**



**Photodissociation** 



#### **Thermal Reaction**



L form

R form

#### **Proton Transfer**

### Nonadiabatic Dynamics in Electrochemistry, Materials, Physics, and Biology



**Charge Transfer** 

MeO

**Energy Transfer** 

**Nonradiative Decay** 



**Charge Transport** 



**Singlet Fission** 



#### **Auger Relaxation**

### Mixed Quantum-Classical Description of Nonadiabatic Dynamics



Mean field (MF):



Ehrenfest Dynamics Ehrenfest, Z. Phys. 45, 445 (1927)

more widely used in physics

#### Surface hopping (SH):



Fewest Switches Surface Hopping Tully, J. Chem. Phys. 93, 1061 (1990)

#### more widely used in chemistry

The treatment of nuclear dynamics is different in MF and SH approaches,

## Advantages of Ehrenfest Mean Field (EMF)



#### Clear physical picture:

The classical part experiences average forces from the quantum part

$$|\Psi(\mathbf{r},\mathbf{R},t)\rangle = |\psi(\mathbf{r},t)\rangle|\chi(\mathbf{R},t)\rangle \exp\left[\frac{i}{\hbar}\int_{0}^{t}E_{0}(t')dt'\right]$$

Representation independence:

 $\frac{\partial |\psi\rangle}{\partial t} = \frac{\mathbf{H}|\psi\rangle}{i\hbar} \qquad m_i \ddot{x}_i = -\frac{\partial \langle \psi | \mathbf{H} |\psi\rangle}{\partial x_i}$ 

Diabatic and adiabatic representations give the same results

#### High efficiency:

Only one trajectory is used to describe the classical dynamics, and thus the statistical error is minimized

### **Limitations of Ehrenfest Mean Field**





Feedback between the electron and nuclei is not properly described and thus the simulation results are not always reliable

### Working Equations of Fewest Switches Surface Hopping (FSSH)





Surface hopping characterize the different motion on different surfaces Adiabatic representation is a natural and preferred choice for surface hopping

### **Beyond Traditional EMF and FSSH**





J. Chem. Phys. 134, 244116 (2011) J. Phys. Chem. Lett. 3, 1888 (2013) J. Chem. Phys. 139, 064316 (2013) J. Chem. Phys. 139, 174109 (2013) J. Phys. Chem. Lett. 5, 713 (2014) J. Chem. Theory Comput. 10, 3598 (2014) J. Phys. Chem. Lett. 6, 3827 (2015) J. Chem. Phys. 143, 191102 (2015) J. Chem. Phys. 148, 104106 (2018) J. Phys. Chem. Lett. 9, 4319 (2018) J. Phys. Chem. Lett. 10, 637 (2019) J. Chem. Phys. 150, 164101 (2019) DOI: 10.1063/1674-0068/cjcp2006098 (2020) DOI: 10.26434/chemrxiv.12820238 (2020)

J. Phys. Chem. Lett. 5, 3345 (2014) Phys. Chem. Chem. Phys. 17, 12395 (2015) Annu. Rev. Phys. Chem. 66, 549 (2015) J. Phys. Chem. Lett. 7, 2100 (2016) WIREs Comput. Mol. Sci. 10, e1435 (2020)

#### **EMF and FSSH share common advantages and limitations**





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### **Quantum Coherence and Decoherence**



#### Schrödinger's cat (thought experiment, 1935):





**Coherence and Decoherence:** 



Coherence decays with time for a quantum system in contact with a classical bath  $\frac{15}{15}$ 

### Decoherence Correction Approaches in Mixed Quantum-Classical Dynamics



#### Existed approaches:

1994, Webster, Rossky, Friesner, SPSH 1995, Schwartz, Bittner, Prezhdo, Rossky, FBD 1997, Prezhdo, Rossky, MF/SH 1999, Fang, Hammes-Schiffer, MDQT\* 2004, Zhu, Jasper, Truhlar, SCDM 2004, Zhu, Nangia, Jasper, Truhlar, CSDM 2005, Zhu, Jasper, Truhlar, LDM, PDDM 2005, Bedard-Hearn, Larsen, Schwartz, MF-SD 2007, Granucci, Persico, EBD 2008, Cheng, Zhu, Truhlar, CSDM-D 2010, Granucci, Persico, Zoccant, ODC 2011, Shenvi, Subotnik, Yang, STSH 2011, Subotnik, Shenvi, A-FSSH 2012, Jaeger, Fischer, Prezhdo, DISH 2013, Gorshkov, Tretiak, Mozyrsky, SCMC 2013, Nelson, Tretiak, ID 2014, Akimov, Prezhdo, CPF 2014, Zheng, Truhlar, AAT-CDSM 2016, Martens, CSH 2017, Gao, Thiel, nH-EOM 2018, Ha, Lee, Min, DISH-XF 2019, Martens, QTSH

#### Decoherence time:



**AFSSH:**  $\tau_i^{-1} = \sum_j Z_{ai}^j \Theta \left( Z_{ai}^j \right) \Theta \left[ \left( \delta R_{aa}^j - \delta R_{ii}^j \right) \left( \delta P_{aa}^j - \delta P_{ii}^j \right) \right]$ 

### Traditional Interpretation of Decoherence Decay of Mixing





#### Frozen Gaussian approximation:

$$g_{i}(x) = \left(\sqrt{2\pi\sigma}\right)^{-1/2} \exp\left[-\frac{(x-x_{i})^{2}}{4\sigma^{2}} + \frac{ip_{i}(x-x_{i})}{\hbar}\right]$$
$$g_{j}(x) = \left(\sqrt{2\pi\sigma}\right)^{-1/2} \exp\left[-\frac{(x-x_{j})^{2}}{4\sigma^{2}} + \frac{ip_{j}(x-x_{j})}{\hbar}\right]$$
$$\left|\left\langle g_{i} \middle| g_{j} \right\rangle\right| = \exp\left[-\frac{(x_{i}-x_{j})^{2}}{8\sigma^{2}} - \frac{\sigma^{2}(p_{i}-p_{j})^{2}}{2\hbar^{2}}\right]$$

First-order approximation: 
$$x_{i} = x_{i}(0) + \frac{p_{i}(0)}{m}t \qquad p_{i} = p_{i}(0) + F_{i}(0)t$$
decoherence rate
$$\left| \langle g_{i} | g_{j} \rangle \right| \approx c \exp\left\{ -\left[ \frac{\left(x_{i}(0) - x_{j}(0)\right)\left(p_{i}(0) - p_{j}(0)\right)}{4\sigma^{2}m} + \frac{\sigma^{2}\left(p_{i}(0) - p_{j}(0)\right)\left(F_{i}(0) - F_{j}(0)\right)}{\hbar^{2}} \right] t \right\}$$

Heller, J. Chem. Phys. 75, 2923 (1981) Neria and Nitzan, J. Chem. Phys. 99, 1109 (1993) Zhu, Nangia, Jasper, and Truhlar, J. Chem. Phys. 121, 7658 (2004) Bedard-Hearn, Larsen, and Schwartz, J. Chem. Phys. 123, 234106 (2005) Subotnik and Shenvi, J. Chem. Phys. 134, 024105 (2011)

### **Time-Dependent Schrödinger Equation**

#### Mixed quantum-classical description:

$$i\hbar \frac{\partial \left| \psi(\mathbf{r},t) \right\rangle}{\partial t} = \hat{H}_{e} \psi(\mathbf{r},t) \rangle$$

Fully quantum Description:

$$i\hbar \frac{\partial \left| \psi(\boldsymbol{r}, \boldsymbol{R}, t) \right\rangle}{\partial t} = \hat{H} \left| \psi(\boldsymbol{r}, \boldsymbol{R}, t) \right\rangle$$
$$\left| \psi(\boldsymbol{r}, \boldsymbol{R}, t) \right\rangle = \sum_{i} c_{i}(t) \left| \phi_{i}(\boldsymbol{r}; \boldsymbol{R}) \right\rangle \otimes \left| \chi_{i}(\boldsymbol{R}) \right\rangle$$

#### Hidden requirements:

- 1. There is only one wave packet (WP) on each potential energy surface
- 2. WPs on different surfaces move with similar trajectories



adiabatic bases

 $\left|\psi(\boldsymbol{r},t)\right\rangle = \sum c_{i}(t) \phi_{i}(\boldsymbol{r};\boldsymbol{R}(t))\right\rangle$ 



### **Phase Correction**



#### **Two Gaussian WPs propagate with different momenta:**

$$p_{2} \qquad |\psi(\mathbf{r}, \mathbf{R}, t)\rangle = \sum c_{i}(t) |\phi_{i}(\mathbf{r}; \mathbf{R}_{i}(t))\rangle \otimes |g_{i}(\mathbf{R}, t)\rangle$$

$$|g_{i}(\mathbf{R}, t)\rangle = \frac{1}{(\sqrt{2\pi\sigma})^{1/2}} \exp \left[-\frac{(\mathbf{x} - \mathbf{x}_{i}(t))^{2}}{4\sigma^{2}} + \frac{i}{\hbar} \mathbf{p}_{i}(t) \cdot (\mathbf{x} - \mathbf{x}_{i}(t)) + i\gamma_{i}(t)\right]$$

**First-order Approximation:** 

$$\boldsymbol{x}_{i}(t) = \boldsymbol{x}_{i}(0) + \frac{\boldsymbol{p}_{i}(0)}{m}t \qquad \boldsymbol{p}_{i}(t) = \boldsymbol{p}_{i}(0) + \frac{\boldsymbol{F}_{i}(0)}{m}t \qquad \boldsymbol{\gamma}_{i}(t) = \boldsymbol{\gamma}_{i}(0) + \frac{\boldsymbol{p}_{i} \cdot \boldsymbol{p}_{i} - \boldsymbol{E}}{\hbar}t$$

Effective Hamiltonian:  

$$\frac{g_2(\mathbf{R}_1,t)}{g_1(\mathbf{R}_1,t)} = \exp\left[\frac{i}{m\hbar}\mathbf{p}_1 \cdot (\mathbf{p}_1 - \mathbf{p}_2)\right] \qquad H_{eff} = \begin{pmatrix} \frac{-\mathbf{p}_1(t) \cdot \mathbf{p}_a(t)}{m} & -\frac{i}{m\hbar}\mathbf{p}_a(t) \cdot \mathbf{d}_{12}(t) \\ -\frac{i}{m\hbar}\mathbf{p}_a(t) \cdot \mathbf{d}_{21}(t) & \frac{-\mathbf{p}_2(t) \cdot \mathbf{p}_a(t)}{m} \end{pmatrix}$$

Heller, J. Chem. Phys. 75, 2923 (1981) Shenvi and Subotnik, J. Chem. Phys. 134, 024105 (2011) Shenvi, Subotnik, and Yang, J. Chem. Phys. 135, 024101 (2011) Zhu, Sci. Rep. 6, 24198 (2016)

### **Branching due to Wave Packet Reflection**



Multiple Gaussian WPs on a PES generated due to reflection:



Multiple Gaussian WPs are necessary in certain cases The WP away from the active trajectory needs to be eliminated, which gives "decoherence correction"

Branching due to wave packet reflection cannot be treated as perturbation





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### Branching Corrected Surface Hopping (BCSH)



#### Resetting wavefunction by judgment of trajectory reflection:



no reflection sign  $\{ F_i(t) \cdot p_i(t) \}$  = sign  $\{ F_i(t) \cdot p_i(t+dt) \}$ 

reflection sign  $\{ F_i(t) \cdot p_i(t) \}$  = -sign  $\{ F_i(t) \cdot p_i(t+dt) \}$ 

In two-level systems, when the active or the non-active surface is reflected, the wave function is collapsed to the active state ( $c_a = 1$  and  $c_{na} = 0$ ) In multilevel systems, we classify states into two groups

### **Standard Scattering Models**





Tully, J. Chem. Phys. 93, 1061 (1990) Subotnik and Shenvi, J. Chem. Phys. 134, 024105 (2011) Xu and <u>Wang</u>\*, J. Chem. Phys. 150, 164101 (2019)

### **Major Channel Performance of BCSH**





**BCSH** reproduces the quantum results almost exactly



### **Minor Channel Performance of BCSH**



Decoherence correction is not very important in these models BCSH reproduces the quantum results even for minor channels

### **Two-Level Model Base**



#### 25 combinations of diabatic potential energy surfaces are constructed

Ibele and Curchod, Phys. Chem. Chem. Phys. 22, 15183 (2020) Xu and <u>Wang</u>\*, J. Chem. Phys. 150, 164101 (2019)

### **Two-Level Model Base**



200 two-level models are constructed to build the model base with 4 strengths and 2 widths for the diabatic coupling

Xu and Wang\*, J. Chem. Phys. 150, 164101 (2019)

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### Performance of BCSH in the Model Base





#### **BCSH** shows the highest performance in the 200 diverse models

#### Xu and Wang\*, (to be submitted)

### **Time-Dependent Population and Coherence**



BCSH reproduces both the population and coherence of the quantum dynamics as demonstrated in the DAG model with k = 25



### **Three-Level Model Base**







### Subotnik's Two-Dimensional Model





#### 2D models are still not extensively investigated in the literature

Subotnik, J. Phys. Chem. A 115, 12083 (2011) Colbert and Miller, J. Chem. Phys. 96, 1982 (1992)

### **Performance of BCSH**



#### BCSH obtains almost the exact population distribution of different channels

Subotnik, J. Phys. Chem. A 115, 12083 (2011) Xu and <u>Wang</u>\*, J. Chem. Phys. 150, 164101 (2019) WG UNN

### **Two-Dimensional Two-Level Model Base**





#### BCSH significantly increases the accuracy in 2D systems

Xu and Wang\*, (to be submitted)

### **Time Evolution of Quantum Population**









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### **New Decoherence Time Formula**



#### Traditional Energy-Based Formula

$$\tau_{ai} = \frac{\hbar}{\left|E_a - E_i\right|} \left(1 + \frac{A}{E_{kin}}\right)$$

Decoherence is stronger for larger kinetic energy, which is against the branching picture

Modified Linear Formula

$$\tau_{ai} = \frac{\hbar}{\left|E_a - E_i\right|} \left(A + B \cdot E_{kin}\right)$$

# Hold the basic formalism and uncertainty principle

Zhu, Nangia, Jasper, and Truhlar, J. Chem. Phys. 121, 7658 (2004) Granucci and Persico, J. Chem. Phys. 126, 134114 (2007) Xiao, Xu, and <u>Wang</u>\*, DOI:10.1063/1674-0068/cjcp2006098 (2020)

Modified Exponential Formula

$$\tau_{ai} = A \exp\left(\frac{B \cdot E_{kin}}{|E_a - E_i|}\right)$$

Tune the dependence on nuclear kinetic energy

#### **Parameter Scan**





#### The parameters are obtained in a machine-learning fashion

### **Traditional Decoherence Time Formula**





The optimal parameter gives no decoherence correction

### **Linear Decoherence Time Formula**



Optimal parameter set corresponds to small A and medium B

### **Exponential Decoherence Time Formula**



Optimal parameter set corresponds to small A and large B

### **Average Errors of the Model Base**



Average errors of the two modified formulas are systematically reduced

### SAC Model





Our modified decoherence time formulas work well the SAC model

### **DAC Model**





Our formulas correctly describe the quantum interference effects

### **ECR Model**





Both our formulas capture the correct strength of decoherence





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### **Branching in the Mean Field Picture**





#### Reference Trajectory:

In surface hopping, the reference trajectory is characterized by the active state. In mean field dynamics, it is the mean field trajectory

#### Branching:

Besides the WPs on adiabatic PESs, the mean field WP can be also reflected. When the mean field trajectory cannot describe all WPs, corrections are needed

### Branching Corrected Mean Field BCMF-s and BCMF-w



**₩P**<sub>NRG</sub>

- Propagate the wave function and nuclei with the average forces
- Check whether the adiabatic WPs and the mean field WP are reflected and classify the adiabatic WPs into RG and NRG groups

$$P_{RG} = \sum_{i \in RG} |c_i|^2 \qquad P_{NRG} = \sum_{i \in NRG} |c_i|^2$$

In BCMF-s, choose either RG or NRG, and reset the corresponding wavefunction coefficients

$$c'_{i \in NRG} = 0 \qquad c'_{i \in RG} = c_i / P_{RG}^{1/2} \qquad c'_{i \in RG} = 0 \qquad c'_{i \in NRG} = c_i / P_{NRG}^{1/2}$$

In BCMF-w, run new mean field trajectories for both RG and NRG with modified weights

 $w_{RG} = w_p P_{RG} \qquad \qquad w_{NRG} = w_p P_{NRG}$ 

Obtain the population of each state considering energy conservation



#### **BCMF-s vs. BCMF-w Algorithms**



Converged results of BCMF-s and BCMF-w are intrinsically identical BCMF-s is more efficient in complex systems for long time simulations BCMF-w is more efficient for simpler cases

### **BCMF vs. EMF in Standard Models**





#### **BCMF** outperforms EMF in the six standard models

Xu and Wang\*, J. Phys. Chem. Lett. in press (2020)

#### **SAC Model**





#### BCMF and BCSH work equally well for the SAC model

#### **DAC Model**





#### BCMF and BCSH work equally well for the DAC model

#### **ECR Model**





#### BCMF and BCSH work equally well for the ECR model

#### **DBG Model**





#### BCMF and BCSH work equally well for the DBG model

#### **DAG Model**





#### BCMF and BCSH work equally well for the DAG model

#### **DRN Model**





#### BCMF and BCSH work equally well for the DRN model



### Average Error in the Two-Level Model Base



BCMF and BCSH work equally well in the 200 scattering problems



### All Errors in the Two-Level Model Base



The population errors are significantly reduced in the 20800 channels

### **Error Distribution in the Two-Level Model Base**



BCMF achieves 10-fold of accuracy compared with the traditional EMF 58

### Summary



A novel branching correction has been proposed for mixed quantumclassical dynamics, promising to be applied to nonadiabatic dynamics simulations in chemistry, physics, biology, and material sciences.

- Reliability significantly improved
- Good for multilevel and high dimensions
- Reproduces time and spacial evolution
- Unifies mean field and surface hopping
- Leads to better decoherence time formulas



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# Thank you for your attention!